Deleveraging, deflation and depreciation in the euro area

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Abstract

Economic performance in the post-crisis period has been heterogenous across the euro area. Some economies rebounded quickly after the output collapse in 2009, while others experienced a further decline of economic activity in the context of an extensive deleveraging process. At the same time, inflation has been subdued across the entire euro area, and intra-euro-area exchange rates have hardly moved. We interpret these facts through the lens of a two-country model. We find that deleveraging in one part of the currency union generates deflationary spillovers which cannot be contained by monetary policy, as it becomes constrained by the zero lower bound. As a result, the real exchange rate response becomes muted, and the output collapse—concentrated in the deleveraging economies.

Keywords: Deleveraging, currency union, real exchange rate, zero lower bound, downward wage rigidity, deflationary spillovers, paradox of flexiblity

JEL-Codes: F41, E42

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1 Introduction

After a highly synchronised output collapse in 2009, economic performance in the euro area has been quite heterogeneous. While some economies rebounded rather quickly, others experienced a much more protracted decline. As a result, aggregate euro area output in 2015 is still below its 2008 level. As some of the most adversely affected countries enjoyed a credit-fuelled expansion prior to the crisis, the post-crisis slump is, at least in part, the result of an ongoing deleveraging process (Martin and Philippon, 2014; Reinhart and Rogoff, 2014).[1] A second distinct feature of the post-crisis period is that inflation has been subdued, not only during the initial downturn, but also during the subsequent years. Moreover, this holds throughout the euro area, including in those economies that have weathered the crisis relatively well. As a consequence, intra-euro-area real exchange rates have hardly moved during the post-crisis slump.

In this paper, we ask why—given the heterogenous economic performance across the euro area after 2009—there is no significant adjustment of intra-euro-area real exchange rates. In our analysis we distinguish between a group of "stressed" economies within the euro area and benchmark their economic performance against an aggregate of euro area economies which did relatively well in the years after 2009. We seek to understand the dynamics of the real exchange rate between these two regions—or, rather, the absence thereof. As both regions are part of the euro area, they lack a flexible exchange rate which—according to the received wisdom—may ease the adjustment of relative prices, if nominal wages or prices are inflexible to adjust (Friedman, 1953).[2] We thus explore the extent to which nominal rigidities can account for the sluggishness of the real exchange rate.

However, intra-euro-area exchange rate movements reflect price level dynamics not only in the stressed economies, but throughout the entire euro area. In fact, the rather strong real exchange rate of the stressed economies is also a result of deflationary pressures observed in the non-stressed economies. In this regard, two observations are important. First, the stressed economies account for a sizeable fraction of the euro area. A severe enough deleveraging process in the stressed economies may thus generate deflationary spillovers, provided monetary policy is unable to contain it. Second, during much of the post-crisis slump monetary policy has indeed been constrained by the zero lower bound. In this environment, deflationary pressure may push up real interest rates and induce a "perverse" response of real exchange

[1] For an early account of how deleveraging in the global banking sector helped the financial crisis in the United States to morph into a global crisis, see Kollmann et al. (2011).

[2] Kollmann (2001) and Monacelli (2004) perform analyses of how nominal rigidities impact real exchange rate volatility under flexible and fixed exchange rates in small open economies. Broda (2004) provides evidence for developing countries in support of the received wisdom.

rates, as observed by Cook and Devereux (2013) for the case of flexible exchange rates.

We also explore this possibility in our analysis. It is based on a model of a currency union which consists of two countries, possibly different in size. Each country specialises in the production of specific intermediate goods. The real exchange rate fluctuates with the relative price of these goods, because goods markets are imperfectly integrated. In the domestic economy, following Eggertsson and Krugman (2012), we distinguish between patient and impatient households and assume that the latter face a borrowing constraint. In the foreign economy, all households are assumed to be patient. In both countries goods prices are flexible, but wages are downwardly rigid as in Schmitt-Grohé and Uribe (2015). Monetary policy aims at stabilising the average inflation rate in the union, but may be constrained by the zero lower bound on nominal interest rates.

We use the model to study the effects of a deleveraging shock in the domestic economy, which is the result of a permanent tightening of the borrowing constraint. The size of the domestic economy is crucial for how this shock plays out. To illustrate this, we first consider two polar cases for which we are able to obtain analytical results. In one case, we let the mass of the domestic economy in the union approach zero. Thus we are effectively looking at a small open economy, such that the deleveraging shock neither has an effect on rest of the union nor on monetary policy. In this case, we find, in line with the received wisdom, that the real exchange rate depreciates in response to the shock and more so, the more flexible wages are in the domestic economy.

However, once we turn to the alternative polar case, in which we let the size of the domestic economy approach unity, a more nuanced picture emerges. Precisely, results depend on whether monetary policy becomes constrained as a result of the shock. The real exchange rate depreciates if monetary policy is unconstrained, irrespectively of the degree of wage rigidity. Instead, the real exchange rate may appreciate, once monetary policy becomes constrained by the zero lower bound. Intuitively, in this case the shock generates deflationary pressure across the entire monetary union. Whether there is appreciation or deprecation depends on specific modelling assumptions. Yet, because of deflationary spillovers, the real exchange rate response is generally muted at the zero lower bound.

We also conduct a quantitative analysis in order to capture important aspects of the post-crisis slump in the euro area. Specifically, we assume that the domestic economy from where the shock originates accounts for just over a third of the euro area. In this case we find that monetary policy ends up at the zero lower bound for plausible parametrisations of the model. Domestic output and prices decline, as do prices in the entire monetary union. Importantly, the decline in prices in the foreign economy implies that foreign output remains close to

full employment. The model thus generates a heterogenous output response across the two countries, while at the same time the real exchange rate remains basically flat. Moreover, a counterfactual analysis reveals that increased wage flexibility in the domestic economy may induce less depreciation in the real exchange rate, rather than more.

This finding relates to the "paradox of flexibility", as established by Eggertsson and Krugman (2012) in a closed-economy setup. Namely, in the face of a large deleveraging shock, increased flexibility may exacerbate the recession as a result of debt deflation à la Fisher (1933). As nominal wages and prices drop more sharply in response to a collapse in demand, the real value of debt increases, which in turn thwarts the initial efforts to reduce the debt. This mechanism is operative in our two-country model, too. More flexibility in the domestic economy may amplify the union-wide effects of the shock. Deflationary spillovers increase, such that there is a tendency for the real exchange rate to appreciate. However, debt deflation does not overturn the received wisdom regarding the benefits of increased wage flexibility, as long as the domestic economy is sufficiently small. In this case, the zero lower bound does not become binding as a result of the shock, and the drop in domestic prices depreciates the real exchange rate, stabilising the economy.

The remainder of the paper is organised as follows. The next section provides a number of basic facts regarding the post-2008 dynamics in the euro area. Section 3 introduces the model. We discuss analytical results for the limiting cases in Section 4. Section 5 presents results obtained from model simulations. A final section concludes.

2 Some facts

In this section we present time-series evidence that highlights important aspects of the post-crisis slump in the euro area, and provides some background for our model-based analysis. Our focus is on two regions, each consisting of a group of countries. The "stressed" region comprises countries where the crisis was particularly severe: we include Greece, Italy, Ireland, Portugal and Spain in this category. The "non-stressed" region consists of Austria, Belgium, Finland, France, Germany and the Netherlands. For each group we aggregate time-series using 2007 GDP weights. The stressed economies make up for roughly 37% of GDP of all the countries in our sample.[3] Our sample, in turn, roughly covers the entirety of the euro area (97% of euro-area GDP).

Figure 1 displays quarterly time-series data for macroeconomic aggregates, covering the period 1999Q1–2014Q4. Dashed lines correspond to data for the euro area as a whole, solid lines correspond to the stressed economies, and dotted lines to the non-stressed economies. The

[3] The 37% figure corresponds to pre-crisis levels. During the crisis, the share falls to around 34%.

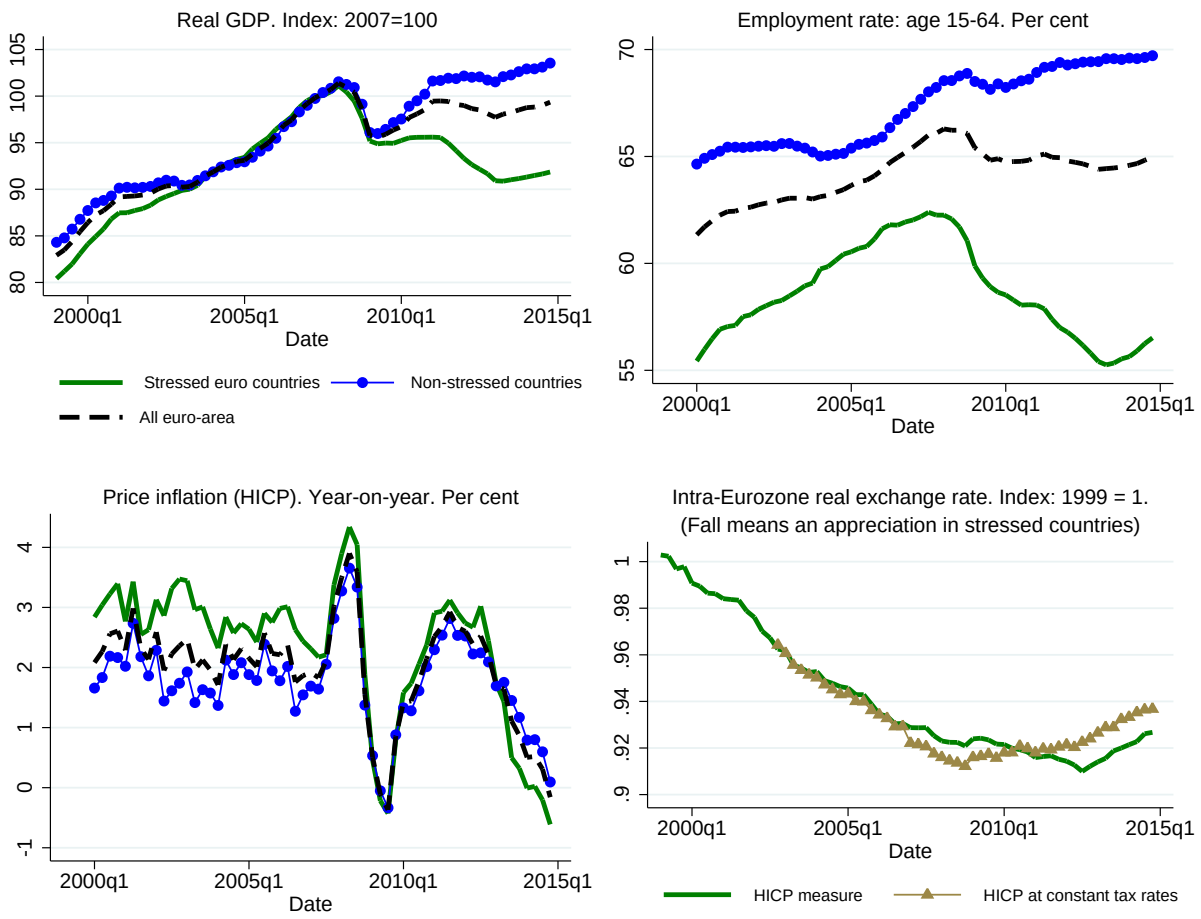

Figure 1: Macroeconomic aggregates, before and after the crisis. Source: Eurostat. Averages are weighted by 2007 nominal GDP. Irish and Finnish tax-adjusted exchange rates series start in 2013 and 2005 respectively; unadjusted data used beforehand.

upper left panel shows real GDP, normalised to 100 in the pre-crisis year 2007. In the run-up to the crisis, output growth was quite synchronised across the two regions. Also, during the early stages of the crisis, GDP fell strongly in both regions. But since 2010 the growth performance has been quite distinct. The decline in GDP in the stressed economies seems to bottom out in 2011–2012, only to decline further afterwards. While there is a small recovery at the end of our sample, GDP is still almost 10 percent below its pre-crisis level. In contrast, GDP recovered relatively quickly in the non-stressed countries, surpassing the pre-crisis level in early 2011. A similar picture emerges when looking at employment data (top right panel). The intra-euro-area real exchange rate is displayed in the bottom right panel. It is normalised to unity in 1999 and defined such that a decline corresponds to an appreciation for the stressed economies. In the years prior to the crisis the real exchange rate appreciated by about 8

percent, but it has changed little since 2008. During first years after the crisis there was a further, if very mild, appreciation and an equally mild depreciation since 2013. The relative price adjustment has been somewhat more pronounced once we consider a measure based the harmonised index of consumer prices at constant tax rates (triangles), but remains muted. Within the euro area, changes in the real exchange rate are the result of differential inflation developments across the two regions, which are shown in the lower left panel. Pre-crisis, inflation in the stressed economies always exceeded inflation in the non-stressed region. On the contrary, during the crisis as well as during the post-crisis period, inflation dynamics have been markedly similar: inflation briefly turned negative in 2009, recovered afterwards but declined again after 2012.

In the model-based analysis that follows, we explore the dynamic adjustment of the real exchange rate to a deleveraging process which takes place in one region of a currency union. That such a process contributed to the post-crisis slump has been suggested by many observers (see, for instance, Martin and Philippon, 2014; Reinhart and Rogoff, 2014) and is hardly controversial in light of the facts. The upper panels of Figure 2 display real credit growth (top row) and credit volumes relative to GDP (middle row) in the euro area, again distinguishing between area-wide developments and those in the stressed and non-stressed economies. In the left and right panels we show private and household debt, respectively.

We observe that prior to the crisis, the stressed economies experienced a particularly rapid expansion of credit. In fact, their real credit growth averaged around 13% per year, roughly triple the corresponding rate in the non-stressed economies. The advent of the crisis in 2008–09 coincides with a collapse in credit growth. After zero credit growth in 2009, it turned negative in the stressed economies and recovered somewhat in the non-stressed economies. In the former, it is still negative at the end of our sample. Credit volumes thus declined considerably during the crisis, but also in the post-crisis period—as far as the stressed economies are concerned. Expressed relative to GDP, overall private credit (household debt) peaked at some 170 (65) percent of GDP in the stressed economies in 2009. Since then, the decline in credit relative to GDP has been muted by the sizeable decline in GDP.

In the bottom panels of Figure 2 we show the dynamics of interest rates. Specifically, we compute the difference of private-sector lending (left panel) and mortgage (right panel) rates relative to the ECB main refinancing rate. The spread on mortgages has been higher in the stressed economies throughout the entire sample period. There is, however, a marked widening in the gap between stressed and non-stressed economies after 2010, reaching some 2 percentage points. The picture is less clear-cut for the spread on private sector lending rates. However, private sector spreads also tightened by considerably more in the stressed

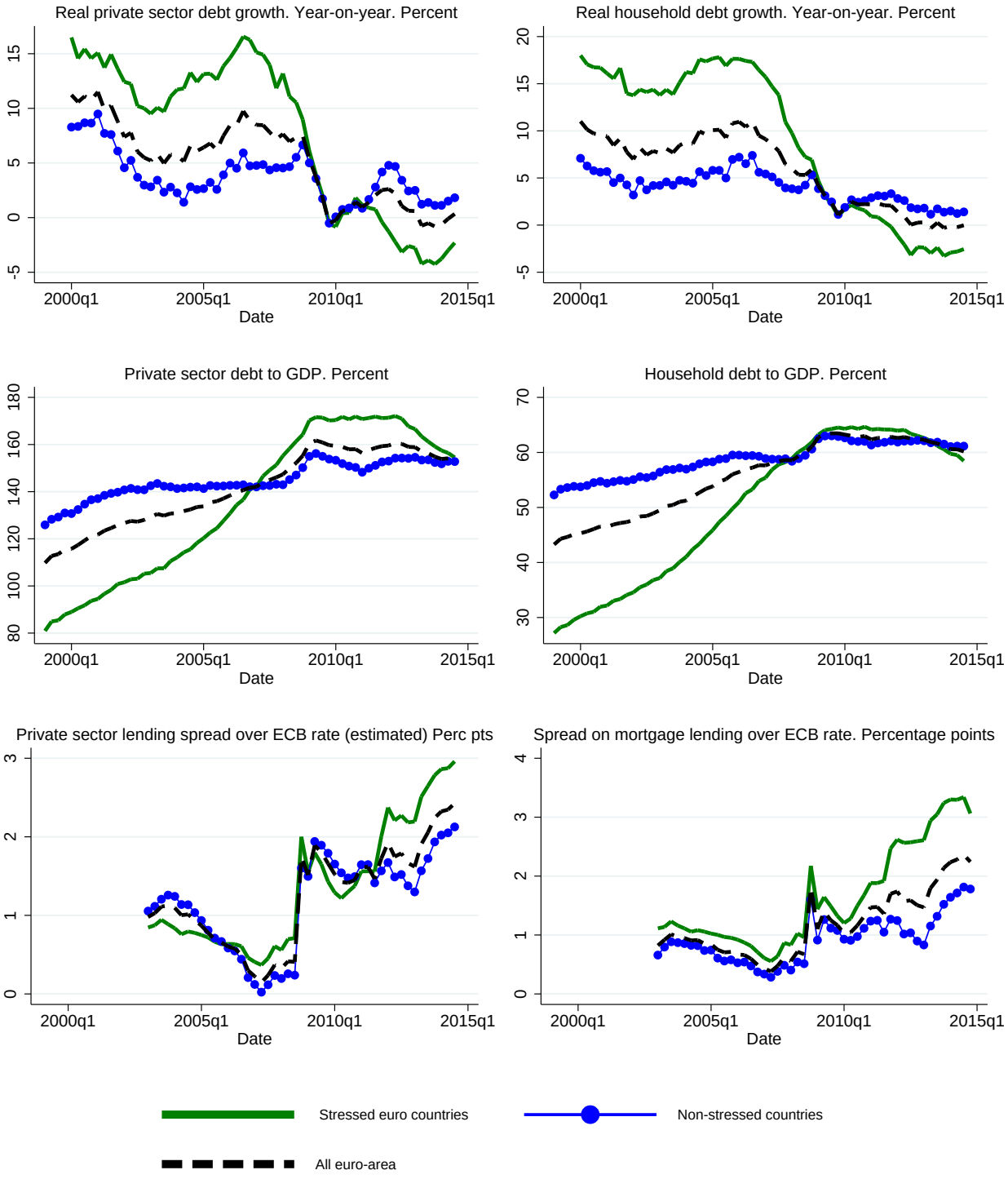

Figure 2: Debt and spreads, before and after the crisis. Sources: Eurostat, ECB and BIS. Averages are weighted by 2007 nominal GDP. Irish household debt data estimated before 2001Q4.

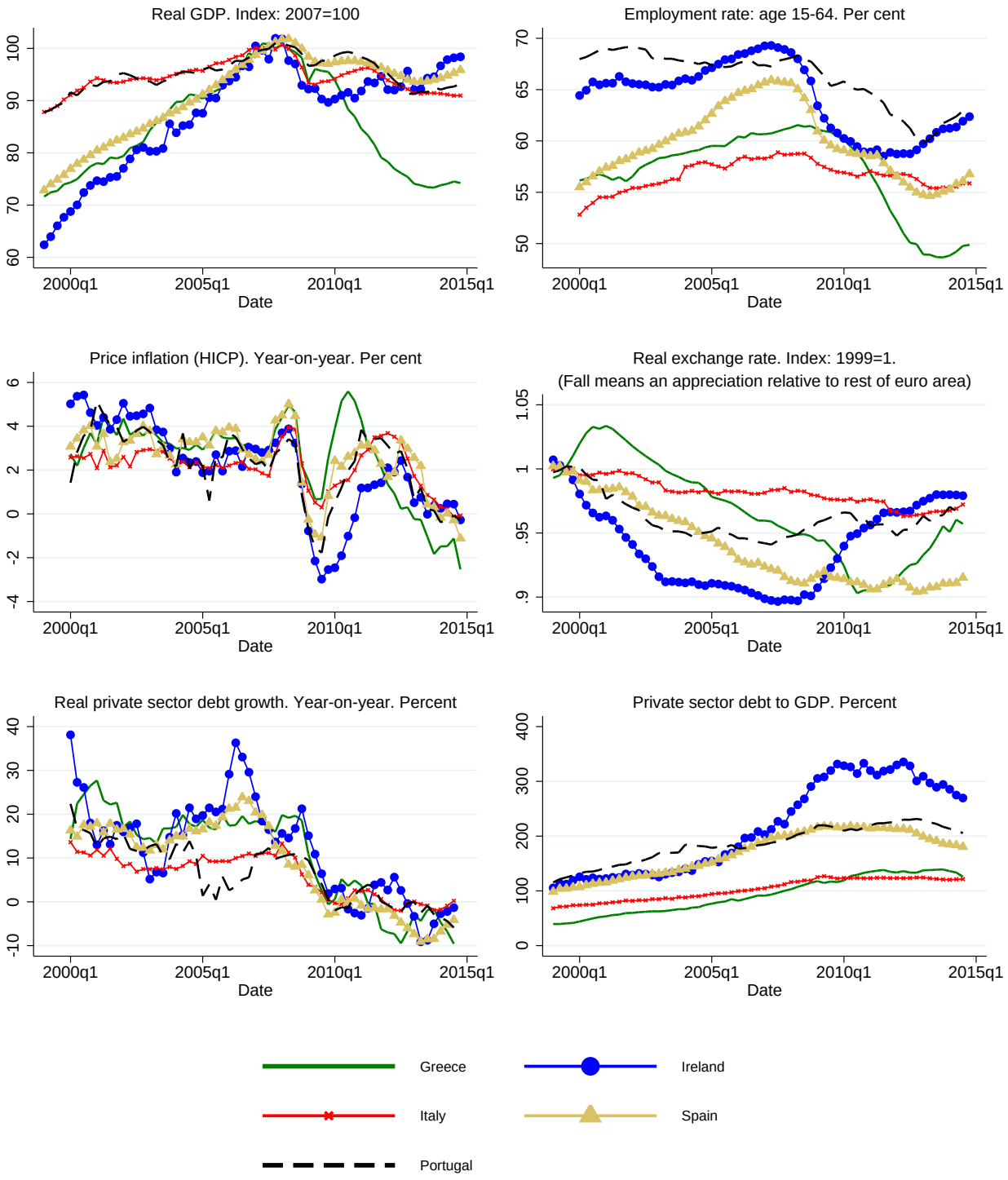

Figure 3: Individual developments among stressed countries. Sources: Eurostat and BIS.

economies in the post-crisis period.

Figure 3 shows the macroeconomic and private sector debt developments in each of the "stressed" countries. As we might expect, the disaggregated picture is more nuanced, with somewhat different dynamics from country to country. Ireland has had the largest lending boom, and also the most significant real exchange rate adjustment. Italy, on the contrary, did not experience much of a boom in lending, or a significant real exchange rate appreciation before the crisis. Greece, in turn, shows the most dismal performance in terms of post-crisis output and employment.

Despite these differences, we may note the following similarities across the stressed euro area countries. First, by looking at the top two graphs (real GDP and employment), all countries have experienced a prolonged slump, and are yet to fully recover. Second, the middle left graph shows that inflation dynamics have been quite synchronised. Third, from the middle right graph we can see that real exchange rates have not adjusted to the levels of 1999-2000 in any country. And fourth, looking at the bottom two graphs, all countries experienced a lending surge before the crisis and a lending slowdown thereafter, albeit to different degrees. Overall, this suggests that the aggregated picture we have discussed earlier is reasonably representative of developments in the stressed economies.

In sum, a simple inspection of the facts supports the view that the stressed economies of the euro area are experiencing a full-fledged balance sheet recession. A sizeable build-up of debt is followed by a lengthy period of deleveraging, with very adverse consequences for economic activity. Instead, there is a recovery in the non-stressed region. Against this background it may be surprising that inflation is subdued not only in the stressed economies, but in the non-stressed region, too. Equivalently, the lack of real deprecation in the stressed economies relative to the non-stressed economies is puzzling. We will investigate this issue further by means of a model-based analysis.

3 The model

Our analysis is based on a simple two-country model of a currency union. Countries specialise in the production of specific goods which are traded across countries. Good market integration is incomplete, however. Countries may differ in size and we assume "Home" makes up a mass $[0, n)$ of the total union population, where $n \in (0, 1)$. Each country is populated by a unit mass of agents, who supply labour inelastically to domestic firms. Within Home we distinguish between households with high and low discount factors as in Eggertsson and Krugman (2012). We refer to these households as "savers" and "borrowers" respectively. The rest of the union ("Foreign") is populated by savers only. Savers in one country can trade a nominally non-

contingent bond vis-à-vis the savers in another country. Savers in Home can, in addition, lend funds to domestic borrowers. Prices are fully flexible, but downward adjustment of nominal wages is restricted as in Schmitt-Grohé and Uribe (2015).

3.1 Households

In Home, borrowers account for a fraction $\chi \in (0,1)$ of the population. They are less patient than savers, which account for the rest. A typical saver in Home maximises

$$\max_{\{C_t^s\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta^s)^t \ln(C_t^s)$$

subject to

$$P_t C_t^s + R_t^{-1} B_t^s + R_t^{*-1} D_t = W_t L_t + B_{t-1}^s + D_{t-1} \tag{3.1}$$

and a no-Ponzi constraint. Here $\beta^s < 1$ is the discount factor which exceeds the discount factor of the borrower: $\beta^s > \beta^b$. C_t^s denotes savers' per capita consumption and P_t is the consumer price level in Home. Savers earn rate R_t by lending to borrowers at home (B_t^s), and rate R_t^* by saving abroad (D_t). All debt and interest rates are denominated in nominal terms. W_t is the nominal wage rate in Home, and L_t are hours worked. Optimality requires the following Euler equation to hold

$$(C_t^s)^{-1} = \beta^s R_t \left(C_{t+1}^s\right)^{-1} \frac{P_t}{P_{t+1}}, \tag{3.2}$$

as well as a transversality condition. The absence of arbitrage possibilities between domestic and foreign assets requires that

$$R_t = R_t^*. \tag{3.3}$$

In turn, a typical borrower maximises

$$\max_{\{C_t^b\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta^b)^t \ln(C_t^b)$$

subject to

$$P_t C_t^b + B_{t-1}^b = R_t^{-1} B_t^b + W_t L_t \tag{3.4}$$

$$B_t^b \leq \bar{B}_t. \tag{3.5}$$

Here, B_t^b denotes nominal debt vis-à-vis the savers which may not exceed an exogenous, potentially time-varying, debt limit $\bar{B}_t$. First order conditions imply that (3.5) holds with

equality at all times.[4] Aggregate consumption in Home is given by

$$C_t = (1-\chi)C_t^s + \chi C_t^b. \tag{3.6}$$

In the rest of the union, all households are savers and we suppress the superscript s for simplicity. The objective is

$$\max_{\{C_t^*\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta^s)^t \ln(C_t^*)$$

subject to

$$P_t^* C_t^* + R_t^{*-1} D_t^* = W_t^* L_t^* + D_{t-1}^*, \tag{3.7}$$

where stars denote variables in the rest of the union. First order conditions imply

$$(C_t^*)^{-1} = \beta^s R_t^* \left(C_{t+1}^*\right)^{-1} \frac{P_t^*}{P_{t+1}^*}. \tag{3.8}$$

We allow for home bias in consumption in both countries, and the elasticity of substitution between domestic and imported goods is unity. Specifically, aggregate consumption is a composite of two goods in both countries:

$$C_t = \frac{C_{H,t}^{\lambda} C_{F,t}^{1-\lambda}}{\lambda^{\lambda}(1-\lambda)^{1-\lambda}}, \qquad C_t^* = \frac{C_{H,t}^{\lambda^*} C_{F,t}^{1-\lambda^*}}{\lambda^{*\lambda^*}(1-\lambda^*)^{1-\lambda^*}},$$

where $\lambda = 1-(1-n)\omega$ and $\lambda^* = n\omega$ (as in De Paoli, 2009). Here, $C_{H,t}$ is the locally produced good, $C_{F,t}$ is the good produced in the rest of the union, and ω measures the degree of home bias in consumption, which we assume is symmetric across countries. The local good sells at price $P_{H,t}$, while the foreign good sells at price $P_{F,t}$. Expenditure minimisation implies

$$P_t = P_{H,t}^{\lambda} P_{F,t}^{1-\lambda}, \qquad P_t^* = P_{H,t}^{\lambda^*} P_{F,t}^{1-\lambda^*}, \tag{3.9}$$

that is, the consumer price indices domestically and abroad are a weighted average of the producer prices of the two goods.

Furthermore, we define the real exchange rate Q_t as the price of foreign consumption in terms of domestic consumption,

$$Q_t = \frac{P_t^*}{P_t}, \tag{3.10}$$

such that an increase in Q_t indicates a depreciation of Home's real exchange rate.

[4] More precisely, optimality requires that $\left(C_t^b\right)^{-1} \geq \beta^b R_t \left(C_{t+1}^b\right)^{-1} \frac{P_t}{P_{t+1}}$, holding with equality whenever $B_t^b < \bar{B}_t$, and requiring $B_t^b = \bar{B}_t$ whenever holding with strict inequality. In the steady state of the model as well as during the deleveraging phase, the latter case always obtains, such that we omit this case distinction from the main text.

3.2 Firms

Firms operate in competitive goods and labour markets. They maximise profits $P_{H,t}Y_t - W_t L_t$ in Home, $P_{F,t}Y_t^* - W_t^* L_t^*$ in Foreign, subject to

$$Y_t = L_t, \qquad Y_t^* = L_t^* \tag{3.11}$$

respectively, and their first order conditions imply

$$P_{H,t} = W_t, \qquad P_{F,t} = W_t^*. \tag{3.12}$$

As in Schmitt-Grohé and Uribe (2015), the labour market is characterised by downward nominal wage rigidity. In each period, a maximum of $\bar{L}$ hours can be sold to firms

$$L_t \leq \bar{L}, \qquad L_t^* \leq \bar{L} \tag{3.13}$$

while wages may fall by at most $(1-\gamma)$ in Home, $(1-\gamma^*)$ in Foreign, in proportion to their previous level

$$W_t \geq \gamma W_{t-1}, \qquad W_t^* \geq \gamma^* W_{t-1}^*. \tag{3.14}$$

We require that $1 \geq \gamma > 0$ and $1 \geq \gamma^* > 0$, where $\gamma, \gamma^* \to 0$ characterises flexible wages, and $\gamma, \gamma^* = 1$—full rigidity. The labour markets are closed by complementary slackness conditions of the form

$$(L_t - \bar{L})(W_t - \gamma W_{t-1}) = 0, \qquad (L_t^* - \bar{L})(W_t^* - \gamma^* W_{t-1}^*) = 0, \tag{3.15}$$

which imply that, as long as wages are free to adjust, the economy must operate at potential. Conversely, involuntary unemployment is possible as (3.14) becomes a binding constraint.

3.3 Monetary policy

We assume that monetary policy is characterised by a strict inflation targeting rule, adjusting the nominal interest rate such that area-wide inflation is zero, subject to a zero lower bound constraint. It targets

$$\Pi_t^u = 1 \text{ subject to } R_t \geq 1, \tag{3.16}$$

where $\Pi_t^u = (P_t)^n (P_t^*)^{1-n} / (P_{t-1})^n (P_{t-1}^*)^{1-n}$ is area-wide inflation, and sets

$$R_t = 1 \tag{3.17}$$

if the inflation target can not be reached.

3.4 Market clearing

Goods market clearing requires that the supply of domestically produced goods equals domestic as well as export demand

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-1} \left(\lambda C_t + \frac{\lambda^*(1-n)}{n} Q_t C_t^*\right). \tag{3.18}$$

Equivalently, we require for the Foreign-produced good[5]

$$Y_t^* = \left(\frac{P_{F,t}}{P_t^*}\right)^{-1} \left(\frac{(1-\lambda)n}{1-n} Q_t^{-1} C_t + (1-\lambda^*) C_t^*\right). \tag{3.19}$$

Moreover, asset market clearing requires

$$(1-\chi) B_t^s = \chi B_t^b \tag{3.20}$$

within Home and

$$n D_t + (1-n) D_t^* = 0 \tag{3.21}$$

across the two countries.

An equilibrium is a sequence of endogenous variables $\{Y_t, Y_t^*, L_t, L_t^*, C_t, C_t^*, C_t^s, C_t^b, B_t^s, B_t^b, ...$ $D_t, D_t^*, R_t, R_t^*, P_t, P_t^*, P_{H,t}, P_{F,t}, W_t, W_t^*, Q_t, \Pi_t^u\}$ solving equations (3.1)—(3.21), for given parameters and initial conditions, and exogenous $\{\bar{B}_t\}$.

3.5 Steady state

We assume that initially the economy is in a symmetric steady state: the real exchange rate, consumer and producer price indices are equal to unity, $P_H = P = P^* = P_F = 1$, which from (3.12) implies that $W = W^* = 1$. Moreover, we let $Y = Y^* = \bar{L}$ and $C^* = Y^*$. This implies $C = Y$ from equations (3.18) and (3.19). We obtain $R = R^* = 1/\beta^s$ from equations (3.2) and (3.8). Borrowers are up against the borrowing constraint, hence $C^b = Y - (1-\beta^s)\bar{B}$. Savers in Home consume $C^s = Y + (1-\beta^s)(B^s + D)$, and savers in Foreign $C^* = Y^* + (1-\beta^s)D^*$. This combined with the fact that $C^* = Y^*$ yields $D^* = 0$, and from (3.21) $D = 0$. That is, net foreign assets must equal zero in the initial steady state. Note that the economy is characterised by non-stationary dynamics, that is, it will generally not revert back to its initial steady state, once it departs from it.[6]

[5] Note that for the limiting case $n \to 0$, $\frac{\lambda^*(1-n)}{n} \to \omega$, and for $n \to 1$, $\frac{(1-\lambda)n}{1-n} \to \omega$.

[6] The economy is non-stationary for two reasons. First, international financial markets are incomplete, and second, households are heterogenous. Both imply that the distribution of wealth, both across agents and across countries becomes a state of the economy, which induces unit-root behaviour in some variables.

4 Relative prices in a crisis

We now investigate how the economy adjusts to a deleveraging shock. More specifically, we consider a one-off tightening of the debt limit in Home from $\bar{B}_t = \bar{B}^H$ to a permanently lower level $\bar{B}^L$ in time period t. Our setup mimics Eggertsson and Krugman (2012), except that we consider a two-country model and restrict the tightening of the debt limit to take place in one country only. The adjustment will then depend on the size of this country. We illustrate this by first focusing on two limiting cases: $n \to 0$ and $n \to 1$. For these cases we obtain closed-form results, and develop an intuition of the underlying mechanisms. We discuss numerical results for intermediate n in Section 5. Throughout, our main interest is how the real exchange rate in period t responds to the shock.

4.1 Deleveraging in a small union member

If $n \to 0$, Home is effectively a small open economy (see De Paoli, 2009; Galí and Monacelli, 2005). The Home-good consumption weights are $\lambda \to 1 - \omega$ and $\lambda^* \to 0$, respectively. In this case, $P_t^* = P_{F,t}$ from equation (3.9), $Y_t^* = C_t^*$ from (3.19) and $D_t^* = 0$ from (3.21). In other words, the rest of the union resembles a closed economy and is not affected by the deleveraging shock in Home. It therefore remains in the initial steady state during the whole deleveraging process. This has important implications for monetary policy: from (3.16) we obtain $\Pi_t^u = P_t^*/P_{t-1}^* = 1$, such that average inflation in the union is zero, and as a result the nominal interest rate remains unchanged as well: $R_t^{-1} = \beta^s$. We summarise our main result in what follows.

Proposition 1. *Consider the economy defined in Section 3 and let $n \to 0$. Suppose that in period t, the debt limit in Home is unexpectedly and permanently reduced from $\bar{B}^H$ to $\bar{B}^L$. The real exchange rate at time t is then given by*

$$Q_t = P_t^*/P_t = \min\left(\gamma^{\omega-1}, [1 - \eta(\bar{B}^H - \bar{B}^L)/Y]^{\omega-1}\right) \geq 1, \tag{4.1}$$

where $\eta = (\beta^s(1-\omega)\chi)/(1-(1-\omega)\chi) > 0$. Therefore, there is no depreciation ($Q_t = 1$) if wages are completely rigid ($\gamma = 1$), and a greater depreciation, the more flexible wages are, where the upper threshold $[1 - \eta(\bar{B}^H - \bar{B}^L)/Y]^{\omega-1} > 1$ is reached once (3.14) ceases to bind.

Proof. See Appendix. □

Intuitively, the deleveraging shock forces borrowers to cut consumption in order to repay their debts. This reduces aggregate demand and puts downward pressure on prices, resulting in a real exchange rate depreciation, as long as wages are allowed to adjust sufficiently. We now establish a second result.

Proposition 2. *Consider again the economy defined in Section 3 with $n \to 0$. In the period of deleveraging, output, saver consumption, borrower consumption and real wage income all decline strictly less if wages are more flexible (that is, if γ is reduced) up until* (3.14) *ceases to bind, point beyond which they do not vary further. The recession is deepest if wages are completely downwardly rigid* ($\gamma = 1$).

Proof. See Appendix. □

Propositions 1 and 2 establish that wage flexibility and the associated movements in the real exchange rate dampen the response to country-specific shocks, as the received wisdom—going back to at least Friedman (1953)—suggests. Still, it is interesting to analyse this case for the following reasons. First, it will serve as a useful benchmark once we consider $n > 0$. Second, the fact that a real depreciation plays a stabilising role is actually not obvious during a deleveraging recession. The fall in Home prices required to bring about the depreciation increases the real value of debt, giving rise to debt deflation à la Fisher (1933).

To see this, consider how the borrowers respond to the deleveraging shock. When the shock hits, nominal debt of $\bar{B}^H - \beta^s \bar{B}^L$ has to be repaid to satisfy the new, lower, borrowing limit. By rearranging the budget constraint (3.4) as follows

$$C_t^b = -\frac{\bar{B}^H - \beta^s \bar{B}^L}{P_t} + \frac{W_t L_t}{P_t}, \tag{4.2}$$

we see that borrowers' consumption depends on debt repayment as well as on real wage incomes $W_t L_t / P_t$. For a given real income, a lower price level increases real debt and reduces consumption. Still, recall that borrower consumption declines less in general equilibrium with greater wage flexibility and real depreciation (Proposition 2). The reason is that in this case, overall economic activity is higher, which helps sustain the real wage income of the borrowers. First, a weaker real exchange rate crowds in foreign demand for the domestic good. Second, since long-run prices are pinned down by purchasing power parity, a temporary drop in the price level generates expected inflation, which reduces the real interest rate and increases spending by savers.[7] It turns out that these two mitigating factors necessarily outweigh the adverse impact of debt deflation. In other words, while rigidity in wages may rule out debt deflation altogether, the resulting drop in real wage income (which operates via a drop in working hours L_t) depresses borrower consumption all the same—in fact, depresses it by more, the more rigid the wages.

In sum, in the case of a small open economy, a lack of relative price adjustment reflects the presence of nominal rigidities, in our case downwardly sticky wages. We show the adjustment

[7] Thus, implicit in fixed exchange rate regimes is an element of price level targeting, which has been emphasised in previous work (Farhi and Werning 2012 and Corsetti et al. 2013).

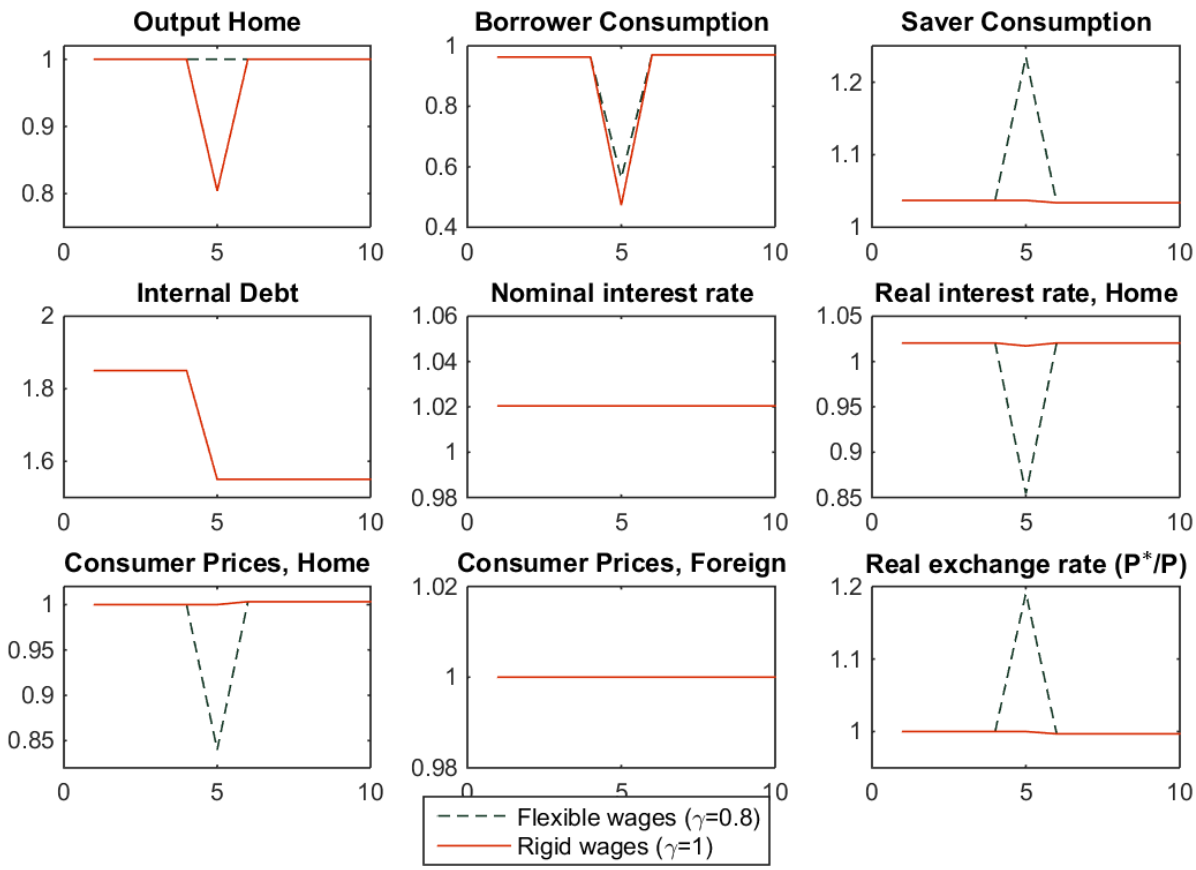

Figure 4: Deleveraging in a small member of a currency union (n $\rightarrow$ 0). Parameter values: $\bar{L} = 1, \bar{B}^H = 1.85, \bar{B}^L = 1.55, \chi = 0.5, \omega = 0.2, \beta^s = 0.98, \beta^b = 0.97$.

dynamics in Figure 4, which contrasts results for the cases of fully rigid ($\gamma = 1$, solid lines) and flexible ($\gamma = 0.8$, dashed lines) wages. We discuss the parameter choices which underly the model simulations in Section 5 below. Importantly, we assume that a deleveraging of 30% GDP is undertaken over one year.[8] Figure 4 echoes our discussion above: if wages are sticky, the exchange rate response is flat, while output collapses. If wages are fully flexible, the exchange rate depreciates strongly, while output remains constant.[9]

The response of consumption also differs across the two scenarios: both saver and borrower consumption are higher under flexible wages, relative to the rigid-wage scenario. This implies, in the context of our model, that welfare is higher under more flexible wages. Galí and Monacelli (2013), in contrast, find that higher wage flexibility may reduce welfare whenever

[8] Here and in the rest of the paper, we refer to deleveraging in terms of nominal GDP before the crisis, which equals 1 in our parametrisation.

[9] In our calibration, $\gamma = 0.8$ provides sufficient flexibility for (3.14) not to bind in the period of deleveraging. A further increase in wage flexibility would then leave results unaltered.

monetary policy seeks to stabilise the exchange rate. This is because Galí and Monacelli (2013) assume a monopolistically competitive labor market and staggered wage setting. As a result, higher wage inflation induces wage dispersion which is detrimental to welfare. We do not consider this possibility. Moreover, recall that in our setup, labour effort has no direct bearing on household utility.

Martin and Philippon (2014) also study a deleveraging shock within a small member country of a currency union. They find their model to perform well in accounting for the dynamics of nominal GDP, employment and net exports in a number of euro area countries during the period 2000–2012. Through counterfactual simulations they explore the role of macro-prudential and fiscal policy in influencing macroeconomic outcomes. Their analysis does not consider movements in relative prices, and in particular real exchange rates, which is the main focus of our paper. Furthermore, focussing on shocks to a small union member—as we do in this section—implies that a deleveraging shock does not generate spillovers on the rest of the union. As we show next, allowing for such spillovers has important implications for the area-wide response to the shock and, hence, for the adjustment of the real exchange rate.

4.2 Union-wide deleveraging

If $n \to 1$, the Home-good consumption weights are $\lambda \to 1$ and $\lambda^* \to \omega$, respectively. We obtain $Y_t = C_t$ from equation (3.18), $P_t = P_{H,t}$ from (3.9) and $D_t = 0$ from (3.21). Thus, Home accounts for almost the entire currency union and behaves like a closed economy. Foreign, in turn, effectively becomes a small open economy. Again, this has important implications for monetary policy: it is entirely geared towards developments in Home, as Foreign has a negligible effect on average union-wide inflation: $\Pi_t^u = P_t/P_{t-1}$ from (3.16).

As we now show, in this case, the real exchange rate response hinges critically on the size of the shock, and the related monetary policy response. We establish that for a large shock, monetary policy becomes constrained by zero lower bound (3.17), and, as a result, the real exchange rate response becomes muted, provided wages in Home are not much more flexible than in Foreign. Moreover, under certain conditions the real exchange rate may in fact appreciate rather than depreciate, reversing the usual dynamics.

Proposition 3. *Consider the economy defined in Section 3 and let $n \to 1$. Suppose that in period t, the debt limit in Home is unexpectedly and permanently reduced from $\bar{B}^H$ to $\bar{B}^L$. The real exchange rate response at time t depends on whether this shock is large enough to push the union to the zero lower bound.*

(a) If the deleveraging shock is small, $\beta^s \bar{B}^H - \bar{B}^L < \underline{\zeta}$, monetary policy is unconstrained by

the zero lower bound and the real exchange rate depreciates. Formally, we have

$$Q_t \ (:= Q_t^{NoZLB}) = \left[(1-\omega)\left(1-\beta^s\left(1-\frac{(1-\chi)Y+\chi\bar{B}^H}{(1-\chi)Y+\chi\bar{B}^L}\right)\right)+\omega\right]^{1-\omega} > 1.$$

(b) *If the deleveraging shock is large, $\beta^s\bar{B}^H - \bar{B}^L > \underline{\zeta}$, the zero lower bound binds in the period of deleveraging. If wages in Home are not much more flexible than in Foreign, $\gamma^*/\gamma < 1+\kappa$, where $\kappa > 0$, then the following inequality holds*

$$Q_t \ (:= Q_t^{ZLB}) = \max\left(\left[\frac{\gamma^*}{\gamma}\right]^{1-\omega}, \left[1-\omega+(1-(1-\omega)\beta^s)\frac{Y_t}{Y}\right]^{1-\omega}\right) < Q_t^{NoZLB}.$$

That is, the zero lower bound generally dampens the real depreciation. Moreover, for a sufficiently large shock, $\beta^s\bar{B}^H - \bar{B}^L > \bar{\zeta} > \underline{\zeta}$, the second part in $\max(\cdot,\cdot)$ *above is below one such that the real exchange appreciates ($Q_t^{ZLB} < 1$), provided wages in Home are less flexible than in Foreign, $\gamma^*/\gamma < 1$.*

Proof. See Appendix. □

The critical values $\underline{\zeta}$, $\bar{\zeta}$ and κ are provided in the appendix, along with the proof of Proposition 3. The solution for Y_t is given in equation (4.4) below; note that $Y_t < Y$, the full-employment output level. The first part of the proposition establishes that country size *per se* does not alter the sign of the real exchange rate response. Intuitively, if monetary policy is not constrained in pursuing the inflation target, it reduces the nominal interest rate sufficiently in response to the deleveraging shock. Average inflation in the currency union remains at zero, because Home inflation is zero ($n \to 1$). Lower interest rates, in turn, raise consumption in Foreign. In the presence of home bias, this pushes up the price level in Foreign. Thus the Home real exchange rate depreciates.

The second part of the proposition shows that if Home is large, the real exchange rate response is generally hampered whenever monetary policy becomes constrained by the zero lower bound. In this case, monetary policy is unable to stabilise Home prices, as this would require pushing nominal interest rates into negative territory. As Home demand collapses, nominal wages (and as a result: prices from (3.12)) decline up to the floor set by downward rigidity, parametrised by γ. This has implications for Foreign, too. As before, Foreign consumption will tend to increase to the extent that monetary policy reduces interest rates. However, there is now a second effect: the demand for Foreign-produced goods falls with Home consumption. This exerts downward pressure on the Foreign price level: there are deflationary spillovers, which dampen the real exchange rate depreciation. In fact, if the shock is large enough ($\beta^s\bar{B}^H - \bar{B}^L > \bar{\zeta}$), and if Foreign wages are more flexible than Home wages,

the Foreign price level may decline more strongly than the Home price level: the Home real exchange rate appreciates.[10]

This finding qualifies a result obtained by Cook and Devereux (2014). They contrast the effects of a negative demand shock in a two-country model under flexible exchange rates and a common currency. In case the zero lower bound binds, they find the Home real exchange rate to appreciate in response to an adverse demand shock, but only if the nominal exchange rate is flexible. Under a common currency, instead, there is no such "perverse adjustment" of the real exchange rate. As we allow for differential degrees of stickiness in the two countries, we find that, for a large enough shock, the real exchange rate response may in fact also reverse its usual pattern under a common currency.

Having established that real exchange rate movements are dampened at the zero lower bound, we now turn to the role of wage rigidity in the adjustment process. It turns out that at the zero lower bound, increasing wage flexibility is actually destabilising, in line with the findings of Eggertsson and Krugman (2012) for the closed economy. We summarise this result in the following Proposition.

Proposition 4. *Paradox of flexibility. Consider the economy defined in Section 3 and let* $n \to 1$. *Assume the zero lower bound is binding in the period of deleveraging* ($\beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$). *In this case, if wages become more flexible domestically (that is, as* γ *is reduced)*

(a) Output, borrower consumption and real wage income all decline strictly more.

(b) The real exchange rate depreciates strictly less (or, if $\beta^s \bar{B}^H - \bar{B}^L > \bar{\zeta} > \underline{\zeta}$, *appreciates strictly more), provided the wage rigidity condition* (3.14) *is not binding in Foreign.*

Proof. See Appendix. □

The reason why more flexibility is harmful is simple: debt deflation. To see this, we again rearrange the borrower budget constraint (3.4) to obtain

$$C_t^b = -\frac{\bar{B}^H - \bar{B}^L}{P_t} + \frac{W_t L_t}{P_t}, \tag{4.3}$$

where we have used that $R_t = 1$ in the period of deleveraging. As before, the shock exerts downward pressure on domestic prices, making real debts harder to repay. And again, the general equilibrium response of real wage income is crucial for how the borrower responds to the shock.

[10] Thus a sufficient condition for the real exchange rate depreciation to be dampened for an intermediate-sized shock, and even to be reversed for a larger shock, is given by $\gamma^* < \gamma$—that is, Foreign wages are more flexible than Home wages.

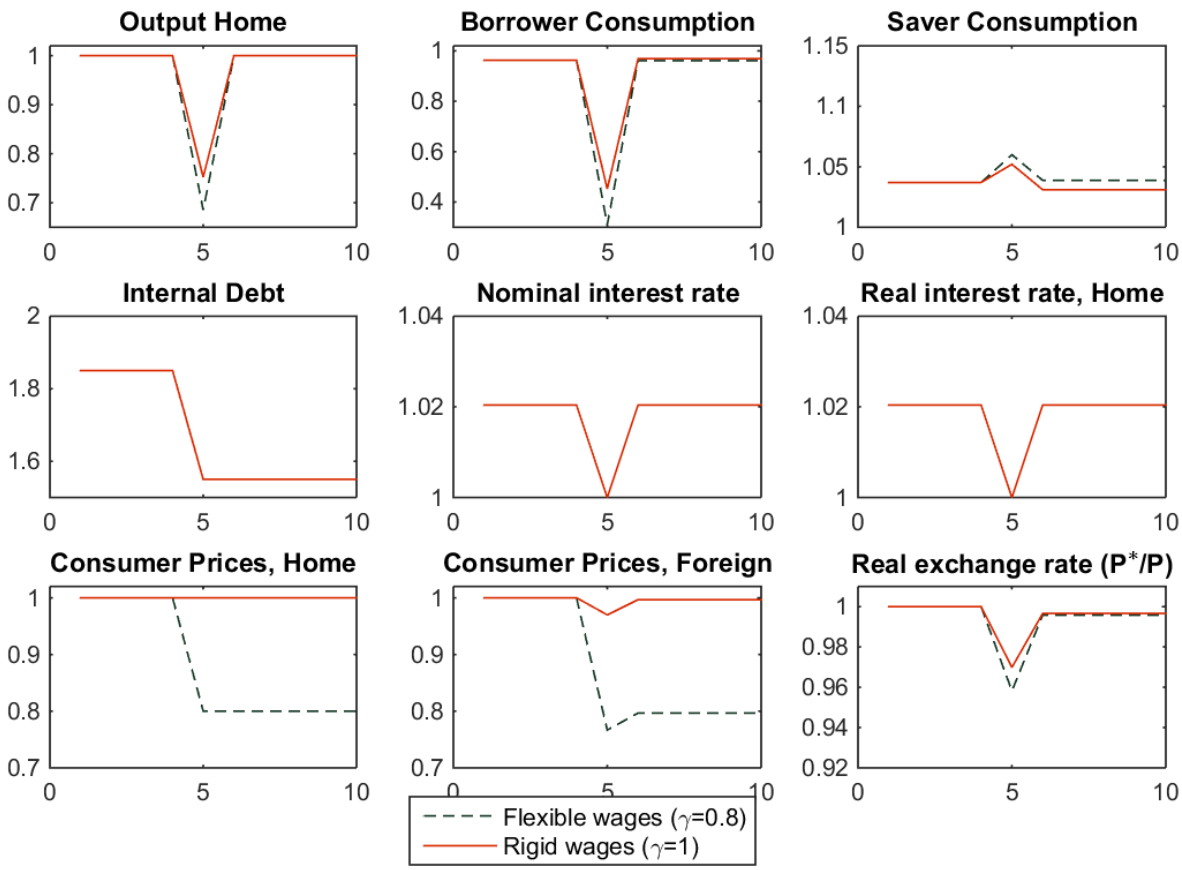

Figure 5: Deleveraging in a large union member (n $\to$ 1). Parameter values: $\bar{L} = 1, \bar{B}^H = 1.85, \bar{B}^L = 1.55, \chi = 0.5, \omega = 0.2, \beta^s = 0.98, \beta^b = 0.97, \gamma^* = 0.7$.

As $n \to 1$, real wage income corresponds to domestic output, $W_t L_t / P_t = Y_t$, which follows from combining (3.11) with (3.12), and using that $P_t = P_{H,t}$ (see above). In turn, domestic output in the period of deleveraging solves[11]

$$Y_t = (\beta^s)^{-1} \left[Y - \frac{\chi}{1-\chi} \left(\frac{\beta \bar{B}^H - \bar{B}^L}{\gamma} \right) \right] < Y. \tag{4.4}$$

The second part of this expression is negative, and more so, the more flexible the domestic wages (the lower the γ). That is, in contrast to the case of $n \to 0$ analysed above, both economic activity and real incomes now decline with more flexible wages.

The intuition here is that while lower prices still increase the real value of debts, they no longer stimulate economic activity by crowding in foreign demand or by lowering the real interest rate. For foreign demand, at the zero lower bound, deflationary spillovers imply that

[11] See the appendix, the proof of Proposition 4.

the real exchange rate response is dampened (Proposition 3).[12] Real interest rates could fall because of either a cut in nominal rates, or expected inflation. But the zero lower bound implies that nominal interest rates are stuck at zero, and under inflation targeting, a drop in prices does not generate expected inflation.

As pointed out in Proposition 3, for a large enough shock, the fall in Home consumption creates large enough spillovers such that Foreign prices fall by more than Home prices, to the extent that the former are more flexible than the latter. Part a) of Proposition 4, in turn, states that these spillovers can only *increase* as domestic rigidities are reduced. Thus, somewhat paradoxically, more flexible wages in Home may lead to an even larger appreciation of its real exchange rate (part b) of Proposition 4).[13] Figure 5 shows the full adjustment dynamics for precisely such a case, by comparing fully rigid ($\gamma = 1$, solid lines) and somewhat more flexible wages ($\gamma = 0.8$, dashed lines).

Generally speaking, while we have only discussed the limiting case of $n \to 1$ in this section, its insights will carry over to smaller values of n, too. As long as the deleveraging shock is large enough to push the union to the zero lower bound, deflationary spillovers will generally mute the adjustment of relative prices, and may sometimes even reverse them. This lack of relative price adjustment, and the lack of movement in the real interest rate, imply that Home could enter a deep recession even if wages are quite flexible. In the next section, we turn to numerical results in order to see whether this is indeed the case for an empirically relevant value of n.

5 Quantitative analysis

The analytical results established in the previous section show that country size matters for the adjustment dynamics to a region-specific deleveraging process. In 2009, credit growth stalled in most countries of the euro area. However, only the stressed economies of the euro area experienced a full-fledged deleveraging process in the years thereafter (see Section 2). In the following we perform a quantitative assessment of our model and explore to what extent it can account for key features of the post-crisis slump in the euro area. Precisely, we examine the non-linear impulse response to a deleveraging shock in two settings. First, we consider the baseline model introduced in Section 3, where deleveraging takes place over one period. Second, we use a modified version of the model, where borrowers are allowed to deleverage

[12] Incidentally, in the case of $n = 1$, foreign demand would not be crowded in even if the real exchange rate depreciated substantially, given Foreign's negligible size. However, as we turn to $n < 1$ next, we believe the deflationary spillovers are the more relevant intuition.

[13] Note that $\gamma^* < \gamma$, the condition for the real exchange to appreciate if the shock is large (part b) of Proposition 3), is implied if the wage rigidity condition (3.14) is not binding in Foreign—the condition stated in part b) of Proposition 4. See the appendix, the proofs of the two propositions for further details.

Parameter:	n	$\bar{B}^H$	$\bar{B}^L$	β^s	ω	χ	γ
Value:	0.37	1.85	1.55	0.98	0.2	0.5	0.9–0.99

Table 1: Parameters for quantitative analysis: baseline model

gradually, and choose the optimal path of deleveraging over a number of periods.

5.1 Baseline model

We assign parameter values in order to solve the model numerically. The specific values are summarised in Table 1. We set $n = 0.37$ to match the share of stressed countries in euro-area GDP on the eve of the crisis. The deleveraging shock brings the levels of debt down from 185% GDP to 155% GDP, that is, by 30 percentage points of GDP, computed on the basis of total private sector debt in the stressed economies.[14]

The other parameters are standard. Assuming a discount factor β^s of 0.98 for the patient households implies an annual real interest rate of 2% in steady state. We set the home-bias parameter $\omega = 0.2$. Given $n = 0.37$ this implies an import share of 0.12, the average GDP weight of imports from the rest of the euro area in the stressed economies.[15] We set the share of borrowers χ to 0.5, a frequently used value in other studies (e.g. Corsetti et al., 2014). Finally, we vary the parameter γ which captures downward wage rigidities. Specifically, we consider a range of values between 0.9 and 0.99, which is equivalent to maximum wage deflation of between 10% and 1% per year respectively. The high downward wage rigidity implied by $\gamma = 0.99$ is broadly in line with estimates for the euro area (Schmitt-Grohé and Uribe, 2015).

Figure 6 shows the response of this economy to the deleveraging shock, contrasting three scenarios. Solid lines represent our baseline, characterized by high wage rigidities ($\gamma = 0.99$) in Home. In addition, we consider two counterfactuals. The lines with crosses correspond to a scenario where Home rigidities are reduced, but remain above Foreign ($\gamma = 0.95$ vs $\gamma^* = 0.92$). The dashed lines correspond to a case where rigidity in Home is significantly lower, to the extent that Home becomes more flexible than Foreign ($\gamma = 0.9$ vs $\gamma^* = 0.92$).

The deleveraging shock is displayed in the bottom-left panel. Under our baseline scenario, it pushes the economy into a deep, but asymmetric recession: while Home output collapses,

[14] The lower limit $\bar{B}^L$ corresponds to the latest observation in our sample (2014Q3) while the upper limit $\bar{B}^H$ corresponds to the value for the same period implied by a linear trend. As before, in our model nominal GDP equals 1 in the pre-crisis steady state, and a deleveraging shock of 0.3 is equivalent to 30% of pre-crisis nominal GDP.

[15] Source: OECD, Monthly Foreign Trade Statistics, period: 1999–2006.

Foreign output remains unaffected, even though monetary policy becomes constrained by the zero lower bound (first row of Figure 6). The dynamics of consumption also differ. While borrowers reduce consumption to repay their debts, Home savers' and Foreign households' consumption increases (second row of Figure 6), reflecting a reduced real interest rate (bottom-right panel for the case of Home).

The third row shows the responses of variables of particular interest. For the baseline case we find a mild decline of the price level in Home, reflecting the presence of downward wage rigidities. At the same time, Foreign prices decline—if only by little—due to deflationary spillovers. As a result, the real exchange rate remains broadly unchanged in the baseline scenario.

Turning to the counterfactuals, consider first the case of somewhat increased wage flexibility ($\gamma = 0.95$, crossed lines) in Home. In this case the real exchange rate depreciates even less than in the baseline case, an instance of the paradox of flexibility (see Proposition 4 for the $n \to 1$ case). Precisely, increased flexibility deepens the recession in Home because of debt deflation. Deflationary spillovers increase, such that prices in Foreign decline by more to stabilise output in the face of an export collapse.

Under our second counterfactual, wage rigidities in Home are relaxed yet further, to the point where wages are more downwardly flexible than in the rest of the union ($\gamma = 0.9$, $\gamma^* = 0.92$, dashed lines). Following the shock, both countries find themselves against the wage rigidity constraint in (3.15), but because Home wages are more flexible, prices in Home fall by more (third row). The real exchange rate depreciates, more so than in the baseline scenario. Turning to output (top row), Foreign now enters a mild recession, because its prices cannot fall by enough to fully offset the negative demand spillover from Home. Recession in Home is still deep, but is slightly milder than under the baseline scenario (by around one percentage point), due to the more favourable movement in the real exchange rate, as well a lower real interest rate. The fall in union-wide output is broadly the same as under the baseline scenario.

On reflection, the results in this section resemble the $n \to 1$ case of Section 4.2 more than they do the $n \to 0$ case of Section 4.1. The deleveraging is associated with a large output drop in Home and deflationary spillovers to Foreign, which increase in size with higher wage flexibility. Unlike Home output and prices, the real exchange rate and Foreign output change little in all three of our simulated scenarios.

In the next section, we examine a fuller model which allows us to relate our quantitative results to the dynamics of the post-crisis slump in the euro area. To do this, we allow the deleveraging to take place gradually, and over a longer time period.

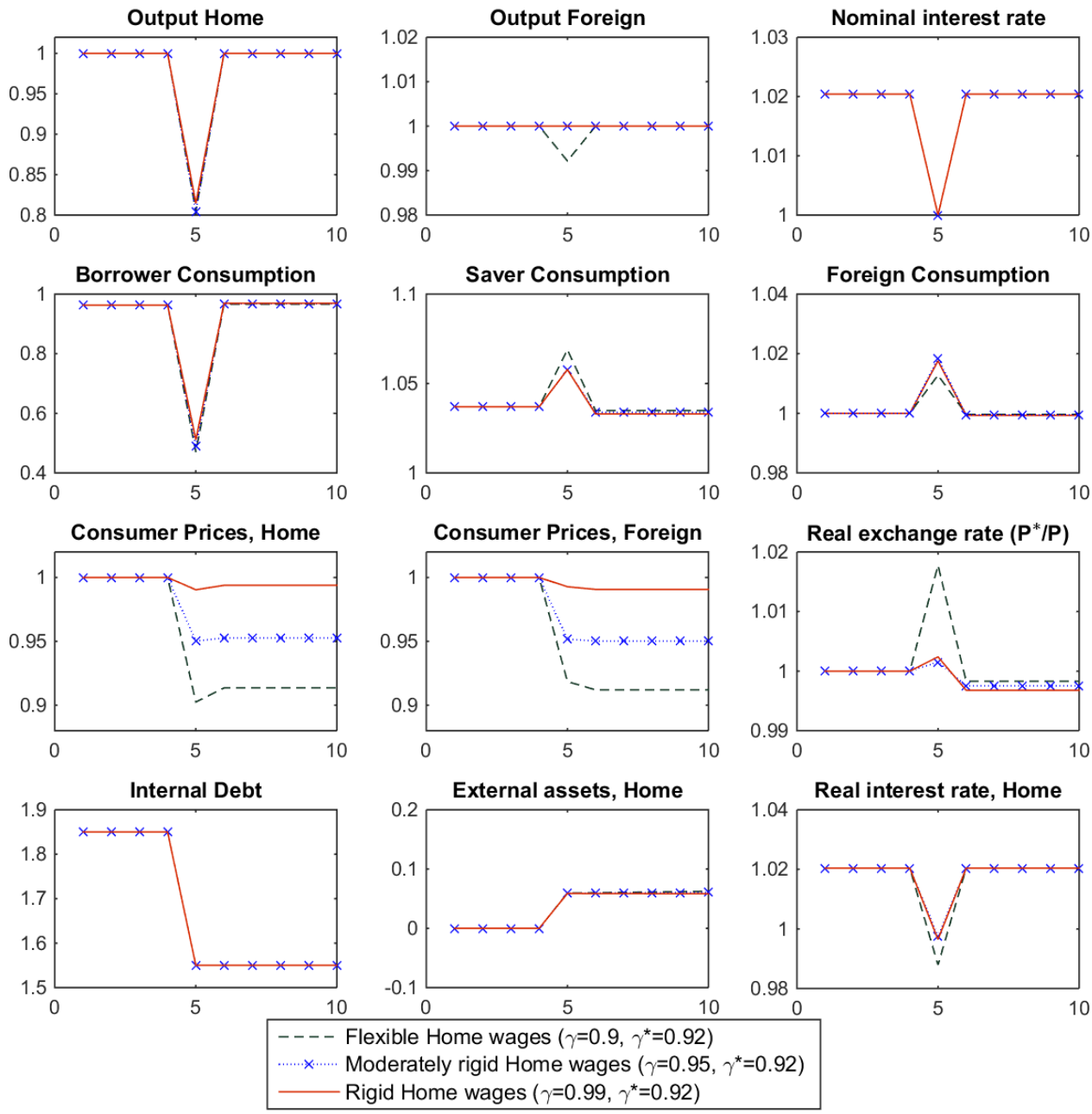

Figure 6: Deleveraging in a medium-sized union member, $n = 0.37$. Baseline model. Parameter values in Table 1.

5.2 Dynamic Deleveraging

Euro-area deleveraging has been taking place for the best part of six years, and the pace of the deleveraging has been quite gradual. To account for this, we modify our model in one key way. Borrowers no longer face a fixed borrowing limit $\bar{B}_t$. Instead, when debts in the economy exceed some level perceived as "safe"—denoted $\bar{B}_{S,t}$—the savers (through financial intermediaries) start charging higher interest rates on borrowing. A deleveraging shock is then a reduction in this safe level of debt, which increases interest spreads and incentivises borrowers to reduce their indebtedness. In this regard we mimic the set-up of Benigno et al. (2014) who consider a closed economy.[16]

Formally, borrowers now maximise

$$\max_{\{C_t^b\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta^b)^t \ln(C_t^b)$$

subject to

$$P_t C_t^b + B_{t-1}^b = (R_t^b)^{-1} B_t + W_t L_t, \tag{5.1}$$

where R_t^b is the gross borrowing rate determined according to

$$R_t^b = R_t \left(B_t^{\text{Agg}} / \bar{B}_{S,t} \right)^{\phi} \tag{5.2}$$

$$R_t^b \geq R_t. \tag{5.3}$$

Note that the spread is determined by aggregate debt, rather than individual debt, even though in equilibrium $B_t^{\text{Agg}} = B_t^b$. Here, ϕ is the elasticity of the gross borrowing rate with respect to excessive debt. As argued in Benigno et al. (2014), one can interpret this set-up as capturing financial intermediation in a very stylized way: banks lend to borrowers at rate R_t^b and pay the savers R_t. The premium for excessive borrowing can be interpreted as a charge for default risk in the presence of asymmetric information, or as compensating for fraud. And the profits of these transactions are distributed to savers, who own the banks.[17]

Optimality requires that borrowers satisfy an Euler equation

$$\left(C_t^b \right)^{-1} = \beta^b R_t^b \left(C_{t+1}^b \right)^{-1} \frac{P_t}{P_{t+1}}. \tag{5.4}$$

Furthermore, as in Benigno et al. (2014), we let $\beta^b \to \beta^s = \beta$, such that in steady state $B^b = \bar{B}_S$ from combining (5.2) and (5.4), and so banks make zero profits.

[16] Our set-up differs only slightly from Benigno et al. (2014), in the following ways. First, we examine two open economies. Second, to improve tractability, the spread is a function of nominal, not real, debt. Last, while under perfect foresight, we solve the model fully non-linearly.

[17] The savers' budget constraint is now given by $P_t C_t^s + R_t^{-1} B_t^s + R_t^{*-1} D_t = W_t L_t + B_{t-1}^s + D_{t-1} + \frac{\chi}{1-\chi} \left(R_t^b - R_t \right) B_t^b$, where the last term are banking profits distributed to savers in a lump-sum manner.

Lastly, for our numerical analysis of the gradual deleveraging model, we assume that the central bank implements its inflation target via a Taylor-type rule of the form

$$R_t = (\beta^s)^{-1} (\Pi_t^u)^{\varphi^\pi} \quad \text{subject to } R_t \geq 1, \tag{5.5}$$

where we assume $\varphi^\pi > 1$.

Parameter:	n	$\bar{B}^H$	$\bar{B}^L$	β	ω	χ	γ	ϕ	φ^π
Value:	0.37	7.4	6.2	0.995	0.2	0.5	0.985–0.99	0.16	1.5

Table 2: Parameters for quantitative analysis: dynamic deleveraging model

We measure periods in quarters, and adjust the model parameters accordingly. All parameter values are listed in Table 2. Values for n, ω and χ are unchanged from before. The quarterly discount factor $\beta = 0.995$ is equivalent to a 2% annualised real interest rate in steady state. Debt limits of 185% and 155% annual GDP translate to 740% and 620% of quarterly GDP respectively. We set the Taylor rule parameter φ^π to the conventional value of 1.5. Wage rigidity occupies a somewhat narrower range, which is more convenient to examine due to the dynamic nature of the model[18]—the values represent maximum wage falls of between 4% and 6% per year. Finally, we set the spread elasticity to 0.16 in order to target a zero-lower-bound episode which lasts 6 quarters.

We now turn to our quantitative analysis. We assume that at time t, the safe debt limit unexpectedly and permanently tightens from $\bar{B}^H$ to $\bar{B}^L$. From (5.2), this opens up a spread between borrowing and lending rates, since the level of debt becomes judged as "excessive": $\bar{B}^H > \bar{B}^L = \bar{B}_{S,t}$, triggering a deleveraging from equation (5.4), until the new safe debt level $\bar{B}^L$ is reached in the long run. The dynamic response to the shock is depicted in Figure 7. As before, we compare the response in a baseline scenario of rigid wages in stressed countries (red solid lines) with a counterfactual scenario of more flexible wages (black dashed lines). Throughout this experiment, we assume that wages in Foreign are flexible enough to support full employment.

From the top row, we can see that the deleveraging shock generates a recession and pushes the union to the zero lower bound. Looking at the mechanism in more detail, the second row shows the associated increase in spreads, which triggers a slow and gradual debt reduction towards the new, lower, safe level. The debt repayment acts as a long-lasting drag on borrower consumption (third row, left), which lowers aggregate demand and generates deflationary

[18] Because of dynamic feedback effects, large changes in wage flexibility can have dramatic effects on model outcomes: they trigger a deflationary spiral, such that a stable model solution does not exist.

pressures. In the early stages of the adjustment, the central bank is unable to cut interest rates sufficiently in order to offset these pressures, and the economy enters a recession.

Turning to the focus of our experiment, the bottom row depicts the movements in relative prices. In every period of the recession, Home wages fall by the maximum amount permitted by the rigidity constraint, which leads to a fall in Home prices. Under the baseline scenario, Foreign prices fall slightly due to the deflationary spillovers, dampening the movement in the real exchange rate, but the real exchange rate still depreciates.

In the counterfactual scenario of more flexible wages, however, the price adjustment is much more pronounced (bottom row, dashed lines). Home prices fall by more, which triggers a sort of an adverse deflationary spiral. Lower prices force borrowers to reduce consumption further (third row, left). At the same time, expected deflation raises real interest rates (second row, right), such that even savers cut back on consumption (third row, middle). This in turn reduces real incomes throughout the Home economy and deepens the recession. The large negative demand spillover pushes down Foreign prices to the extent that the real exchange rate appreciates at first, further depressing real incomes and exacerbating the deleveraging in Home. The paradox of flexibility is in full force, and the recession is deeper than under the baseline scenario of more rigid wages.

The discussion above supports the conclusions of our analytical model in Section 4, and the one-period deleveraging experiment in Section 5. It adds two further qualifications: first, the dynamic nature of deleveraging means the recession and zero lower bound episodes can last longer, and the reduction in debt levels can be very gradual—indeed much longer than the duration of the recession. Second, because higher levels of debt persist for longer, and some of the deflation is anticipated, the adverse effects of lower prices on real interest rates and real debt levels seem to be more pronounced.

Even though the model in this section remains fairly stylised, the impulse responses for our baseline scenario match a number of facts which characterize the post-crisis slump in the euro area (Section 2). Just like the stressed euro-area economies, there is a deep recession in the Home country, triggered by a long and gradual deleveraging process. The recession persists for several periods and pushes the union to the zero lower bound. Prices in Home and Foreign move in a synchronised manner, which means that the real depreciation is small, around 2.6% at its peak. This is still somewhat larger than the "raw" 0.6% real depreciation we observe for the stressed euro countries in the data, but broadly in line with the 2% depreciation calculated using tax-adjusted price data.[19] The fall in Foreign price level helps the rest of the union remain at close to full employment, so whilst prices move together, the output performance

[19] Figures quoted are for 2008Q4—2014Q4.

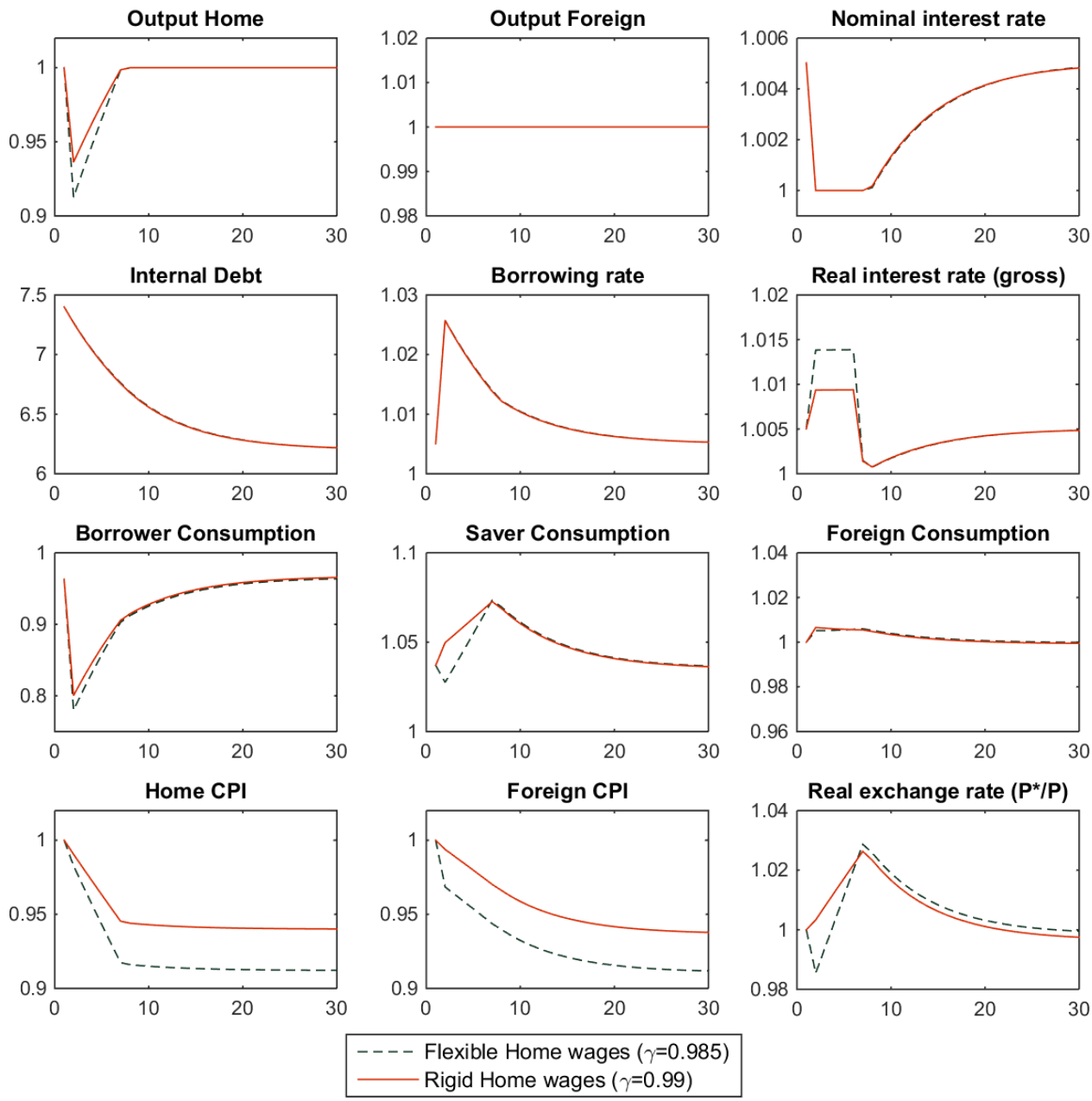

Figure 7: Dynamic deleveraging in a medium-sized union member, $n = 0.37$. Parameter values in Table 2. Interest rates are shown on a quarterly basis.

of the two country groups diverges. Our counterfactual simulation suggests that had wages in stressed economies been more flexible, we would have still seen little relative price adjustment within the euro area. Indeed, the real exchange rate may have even appreciated, deepening the recession.

6 Conclusion

Why—given the heterogenous economic performance across the euro area after 2009—has there been no significant adjustment of intra-euro-area real exchange rates? In this paper, we address the question by setting up a model of a currency union in which one of the two member countries experiences a large deleveraging shock. A key feature of the model is that the size of countries may differ. This allows us to study polar cases for which we are able to solve the model in closed form. In doing so we clarify aspects of the transmission mechanism, which also turn out to be relevant in the quantitative assessment of the model.

In case the domestic economy is small, there are no spillovers from the deleveraging shock into the rest of the union. Also, monetary policy remains unchanged. In this case, in line with much of the received wisdom, the extent of real deprecation is inversely related to the degree of wage rigidities. In the context of our model, this is simply the result of prices fluctuating one-for-one with wages. More flexibility in wages not only makes the real exchange rate more flexible, it also makes the economy more resilient in the face of adverse country-specific shocks. This result is notable, because more wage flexibility also leads to more debt deflation in response to the deleveraging shock. Still, this effect is not strong enough to offset the benefits from increased flexibility for a small member country of a currency union.

A different picture emerges once we consider a large currency union member. Monetary policy lowers interest rates in response to the shock, and reaches the zero lower bound, if the shock is large enough. The output response in the domestic economy and in the rest of the union will generally be different, in particular if there is sufficient wage flexibility in the rest of the union: while domestic output collapses, it may stay at its initial level in the other countries. At the same time there is deflationary pressure across the entire union, and the response of the real exchange rate to the deleveraging shock is dampened if the zero lower bound binds. In fact, the real exchange rate may even appreciate if the shock is large and/or wages are relatively flexible in the rest of the union. More wage flexibility in the domestic economy can also be destabilising and induce an even stronger real appreciation, an instance of the paradox of flexibility, established by Eggertsson and Krugman (2012) in a closed-economy context.

In our quantitative assessment we assume that the domestic economy accounts for 37 percent of the currency union, corresponding to the GDP weight of the stressed economies prior to

the crisis. In response to a deleveraging shock of 30 percent GDP, we find the model able to account for some of the basic facts of the post-crisis slump in the euro area: there is large divergence in terms of output performance, there is deflationary pressure in the entire union, and the real exchange rate response is muted.

We also find that the paradox of flexibility reigns in our calibrated model. In a counterfactual scenario where wages are more flexible, the real exchange rate appreciates and the output loss is even larger than in the baseline case. Hence, we think that our analysis has some relevance for the ongoing policy debate in the euro area. Namely, similarly to Eggertsson et al. (2014), we find that measures geared towards raising the flexibility of the economies hit hardest by the crisis are likely to be ineffective—unless they are accompanied by expansionary monetary policy measures which may make up for the zero-lower-bound constraint on interest rates.

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Appendix

This Appendix presents proofs of Propositions 1—4.

A1 Proposition 1 proof

For $n \to 0$, we can reduce the system of equations (3.1)—(3.21) to the following 7 equations:

$$P_t C_t^s = P_{t+1} C_{t+1}^s \tag{A1.1}$$

$$P_t C_t^b = \beta^s \bar{B}_t - \bar{B}_{t-1} + P_{H,t} Y_t \tag{A1.2}$$

$$C_t = (1-\chi) C_t^s + \chi C_t^b \tag{A1.3}$$

$$(1-\chi)(\beta^s D_t - D_{t-1}) = P_{H,t} Y_t - P_t C_t \tag{A1.4}$$

$$P_t = P_{H,t}^{1-\omega} \tag{A1.5}$$

$$P_{H,t} Y_t = (1-\omega) P_t C_t + \omega Y \tag{A1.6}$$

$$0 = (Y_t - Y)(P_{H,t} - \gamma P_{H,t-1}), \tag{A1.7}$$

as well as two inequalities:

$$Y_t \leq Y \tag{A1.8}$$

$$P_{H,t} \geq \gamma P_{H,t-1} \tag{A1.9}$$

The first equation is derived from the saver's Euler equation (3.2), the second—from the borrower's budget constraint (3.4) (combined with (3.11), (3.12)), and the third—from aggregate consumption (3.6). Equation (A1.4) is the country's budget constraint, obtained by combining the saver's and borrower's budget constraints (3.1) and (3.4), and substituting for nominal incomes from (3.11) and (3.12). The remaining equations are the price index (from 3.9), Home market clearing (from 3.18), and the complementary slackness condition (from 3.15). The inequality in (A1.9) is obtained by combining (3.14) and (3.12). Additionally, we have used that $R_t = (\beta^s)^{-1}$, $P_t^* = P_{F,t} = 1$, and $C_t^* = Y_t^* = Y$.

The perfect foresight solution is a sequence of endogenous variables $\{C_t^s, C_t^b, C_t, Y_t, P_t, P_{H,t}, D_t\}$ that solves equations (A1.1)—(A1.9), given the initial conditions (stated in Section 3.5), and an exogenous path for $\{\bar{B}_t\}$, known in the initial period.

We can solve for real and nominal variables separately. The system in nominal variables can be reduced further to three equations

$$P_t C_t^s = P_{t+1} C_{t+1}^s \tag{A1.10}$$

$$(1-\chi)(\beta^s D_t - D_{t-1}) = (1-\chi) P_{H,t} Y_t - (1-\chi) P_t C_t^s - \chi(\beta^s \bar{B}_t - \bar{B}_{t-1}) \tag{A1.11}$$

$$(1-(1-\omega)\chi) P_{H,t} Y_t = (1-\omega)(1-\chi) P_t C_t^s + (1-\omega)\chi(\beta^s \bar{B}_t - \bar{B}_{t-1}) + \omega Y \tag{A1.12}$$

in three unknowns $(P_t C_t^s), (P_{H,t} Y_t)$ and D_t.

We solve this system forward analytically. The full solution is presented in the proof of Proposition 2. Here, we only present the solution for Home nominal income $(P_{H,t} Y_t)$:

$$P_{H,t} Y_t = Y + (1-\beta^s)\frac{(1-\chi)(1-\omega)}{\omega} D_{t-1} - \eta(\bar{B}_{t-1} - \bar{B}_t),$$

where $\eta = (\beta^s(1-\omega)\chi)/(1-(1-\omega)\chi) > 0$.

In the period of deleveraging, $D_{t-1} = 0$, $\bar{B}_{t-1} = \bar{B}^H$ and $\bar{B}_t = \bar{B}^L$, which yields

$$P_{H,t} Y_t = Y - \eta(\bar{B}^H - \bar{B}^L), \tag{A1.13}$$

Under full employment, $Y_t = Y$, and hence

$$P_{H,t} = 1 - \eta(\bar{B}^H - \bar{B}^L)/Y. \tag{A1.14}$$

If there is unemployment, $Y_t < Y$ and from (A1.7)

$$P_{H,t} = \gamma P_{H,t-1} = \gamma$$

From (A1.9),

$$P_{H,t} = \max(\gamma, 1 - \eta(\bar{B}^H - \bar{B}^L)/Y \leq 1. \tag{A1.15}$$

The inequality holds because $\gamma \leq 1$.

From (3.10) and (A1.5), $Q_t = P_{H,t}^{\omega-1}$, where $\omega - 1 < 0$. Combining this with (A1.15) gives us the real exchange rate formula in Propostion 1:

$$Q_t = P_t^*/P_t = \min\left(\gamma^{\omega-1}, [1 - \eta(\bar{B}^H - \bar{B}^L)/Y]^{\omega-1}\right) \geq 1.$$

A2 Proposition 2 proof

We first present the full solution to the system of equations (A1.10)—(A1.12):

$$\begin{aligned}
P_t C_t^s &= Y + (1-\beta^s)\left(\frac{\chi}{1-\chi}\bar{B}_{t-1} + \frac{1-(1-\omega)\chi}{\omega} D_{t-1}\right) \\
P_t C_t^b &= Y - (1-\beta^s)\left(\bar{B}_{t-1} - \frac{(1-\chi)(1-\omega)}{\omega} D_{t-1}\right) - \frac{\beta^s}{1-(1-\omega)\chi}(\bar{B}_{t-1} - \bar{B}_t) \\
P_{H,t} Y_t &= Y + (1-\beta^s)\frac{(1-\chi)(1-\omega)}{\omega} D_{t-1} - \frac{\beta^s(1-\omega)\chi}{1-(1-\omega)\chi}(\bar{B}_{t-1} - \bar{B}_t) \\
D_t &= D_{t-1} + \frac{\omega\chi}{(1-(1-\omega)\chi)(1-\chi)}(\bar{B}_{t-1} - \bar{B}_t).
\end{aligned}$$

In the period of deleveraging, $D_{t-1} = 0$, $\bar{B}_{t-1} = \bar{B}^H$ and $\bar{B}_t = \bar{B}^L$. Substituting for this gives the following expressions for nominal spending and incomes:

$$\begin{aligned} P_t C_t^s &= Y + (1-\beta^s)\frac{\chi}{1-\chi}\bar{B}^H \\ P_t C_t^b &= Y - (1-\beta^s)\bar{B}^H - \frac{\eta}{(1-\omega)\chi}(\bar{B}^H - \bar{B}^L) \\ P_{H,t} Y_t &= Y - \eta(\bar{B}^H - \bar{B}^L), \end{aligned}$$

where $\eta = (\beta^s(1-\omega)\chi)/(1-(1-\omega)\chi) > 0$, as above.

We can immediately see that nominal spending and incomes are independent of wage flexibility γ. Savers' nominal spending is independent of the deleveraging shock, whilst borrowers' nominal spending and incomes fall proportionately with the deleveraging shock $\bar{B}^H - \bar{B}^L$. Because of this, the smaller the nominal adjustment in prices, the larger the adjustment of real spending and incomes will be, which gives us some intuition for the benefits of higher wage flexibility. We establish this more formally below.

Suppose first that the wage rigidity condition (A1.9) is binding. Then $P_{H,t} = \gamma$ and $P_t = \gamma^{1-\omega}$ from (A1.5). Real spending and output, as well as real incomes $W_t L_t / P_t$, are given below

$$C_t^s = \gamma^{\omega-1}\left[Y + (1-\beta^s)\frac{\chi}{1-\chi}\bar{B}^H\right] \tag{A2.1}$$

$$C_t^b = \gamma^{\omega-1}\left[Y - (1-\beta^s)\bar{B}^H - \frac{\eta}{(1-\omega)\chi}(\bar{B}^H - \bar{B}^L)\right] \tag{A2.2}$$

$$Y_t = \gamma^{-1}\left[Y - \eta(\bar{B}^H - \bar{B}^L)\right] \tag{A2.3}$$

$$W_t L_t / P_t = \gamma^{\omega} Y_t = \gamma^{\omega-1}\left[Y - \eta(\bar{B}^H - \bar{B}^L)\right], \tag{A2.4}$$

and all increase with lower γ, or higher wage flexibility (note that $\omega - 1 < 0$).

When γ is low enough such that the wage rigidity condition in (A1.9) is not binding, from (A1.15), prices are at their lowest level given by (A1.14), output is at full employment level, $Y_t = Y$, and consumption and real incomes are at their maximum level.[20] If γ is lowered further beyond this point, the real variables no longer change. This concludes the proof of Proposition 2.

[20] That is, the maximum when varying wage flexibility and taking all other parameters as given.

A3 Proposition 3 proof

For $n \to 1$, we can reduce the system of equations (3.1)—(3.21) to the following 11 equations:

$$\beta^s R_t P_t C_t^s = P_{t+1} C_{t+1}^s \tag{A3.1}$$

$$P_t C_t^b = R_t^{-1} \bar{B}_t - \bar{B}_{t-1} + P_t Y_t \tag{A3.2}$$

$$C_t = Y_t \tag{A3.3}$$

$$C_t = (1-\chi) C_t^s + \chi C_t^b \tag{A3.4}$$

$$\beta^s R_t P_t^* C_t^* = P_{t+1}^* C_{t+1}^* \tag{A3.5}$$

$$R_t^{-1} D_t^* - D_{t-1}^* = \omega (P_t C_t - P_t^* C_t^*) \tag{A3.6}$$

$$P_{F,t} Y^* = (1-\omega) P_t^* C_t^* + \omega P_t C_t \tag{A3.7}$$

$$P_t^* = P_{F,t}^{1-\omega} P_t^{\omega} \tag{A3.8}$$

$$0 = (R_t - 1)(P_t - P_{t-1}) \tag{A3.9}$$

$$0 = (Y_t - Y)(P_t - \gamma P_{t-1}), \tag{A3.10}$$

$$0 = (Y_t^* - Y)(P_{F,t} - \gamma^* P_{F,t-1}), \tag{A3.11}$$

as well as these inequalities:

$$Y_t \leq Y, \qquad Y_t^* \leq Y \tag{A3.12}$$

$$P_t \geq \gamma P_{t-1}, \qquad P_{F,t} \geq \gamma^* P_{F,t-1} \tag{A3.13}$$

$$R_t \geq 1 \tag{A3.14}$$

The first five equations are the Home savers' Euler equation (3.2), the borrower's budget constraint (3.4), Home goods market clearing (3.18), Home aggregate consumption (3.6), and Foreign saver's Euler equation (3.8) respectively. Equation (A3.6) is the Foreign country's budget constraint, which we obtain by combining the Foreign saver's budget constraint (3.7) with the Foreign goods market clearing condition (3.19), and using the expressions in (3.10), (3.11) and (3.12). The remaining equations are, in order, the Foreign goods market clearing (3.19), Foreign price index (3.9), monetary policy (equations 3.16 and 3.17), and the complementary slackness conditions in Home and Foreign, in (3.15). Inequalities refer to the full employment constraint on output, downward wage rigidity and the zero lower bound. Throughout, we have used $P_t^u = P_t = P_{H,t} = W_t$, and $D_t = 0$ due to the large size of Home.[21] Additionally, we write down a simplified expression for the savers' budget constraint in Home, which is not necessary to compute the solution, but provides a useful stepping stone in parts

[21] $P_t^u = (P_t)^n (P_t^*)^{1-n}$ is the union-wide price level.

of the proof:

$$P_t C_t^s = \frac{\chi}{1-\chi}(\bar{B}_{t-1} - R_t^{-1}\bar{B}_t) + P_t Y_t. \tag{A3.15}$$

The perfect foresight solution is a sequence of endogenous variables $\{C_t^s, C_t^b, C_t, Y_t, P_t, R_t, P_t^*, C_t^*, D_t^*, Y_t^*, P_{F,t}\}$ that solves equations (A3.1)—(A3.14), given the initial conditions (stated in Section 3.5), and an exogenous path for $\{\bar{B}_t\}$, known in the initial period. The solution will depend on whether the union is at zero lower bound or not. We consider each of these two cases in turn.

(a) Outside of zero lower bound

For the remainder of the proof, we adopt the same time notation as in Proposition 3: rather than using the general time subscript t, we denote t as the period of deleveraging, and $t-1$ as the initial steady state. Equation (A3.9) then yields $P_t = P_{t-1} = 1$, and hence $Y_t = Y$ from equation (A3.10): we have zero inflation and full employment in Home during the deleveraging period. At this point, it is helpful to write down the savers' budget constraint (A3.15) at t and $t+1$:

$$C_t^s = Y + \frac{\chi}{1-\chi}\left(\bar{B}^H - R_t^{-1}\bar{B}^L\right)$$
$$C_{t+1}^s = Y + \frac{\chi}{1-\chi}(1-\beta^s)\bar{B}^L$$

where we have used that $Y_t = Y_{t+1} = Y$, $\bar{B}_{t-1} = \bar{B}^H$ and $\bar{B}_t = \bar{B}^L$.

Plugging the above expressions into the savers' Euler equation (A3.1), we obtain an expression for the nominal interest rate

$$R_t = (\beta^s)^{-1}\frac{(1-\chi)Y + \chi\bar{B}^L}{(1-\chi)Y + \chi\bar{B}^H} \tag{A3.16}$$

Intuitively, the central bank cuts the nominal interest rate sufficiently such that the savers consume all of the debt repayments they receive from the borrowers to maintain full employment.

We now turn to the developments in Foreign. In the period before deleveraging $D_{t-1}^* = 0$, and thereafter $D_{t+1}^* = D_t^*$ and $R_{t+1} = (\beta^s)^{-1}$. Also, there is no downward pressure on Foreign prices, and it remains at full employment, $Y^* = Y$.[22] We can then write down a 4x4 equation

[22] One can verify this ex post by checking the real exchange rate formula in (A3.16). Since $Q_t > 1$, $P_{F,t} > 1$ and $P_{F,t} > P_{F,t-1}$. From (A3.11), this implies $Y_t^* = Y$.

system which allows us to solve for the Foreign price level, and hence, the real exchange rate.

$$R_t P_t^* C_t^* = (\beta^s)^{-1} P_{t+1}^* C_{t+1}^* \tag{A3.17}$$

$$R_t^{-1} D_t^* = \omega(P_t C_t - P_t^* C_t^*) \tag{A3.18}$$

$$(1-\beta^s) D_t^* = \omega(P_{t+1}^* C_{t+1}^* - P_{t+1} C_{t+1}) \tag{A3.19}$$

$$P_{F,t} Y = (1-\omega) P_t^* C_t^* + \omega P_t C_t \tag{A3.20}$$

The first equation is Foreign saver's Euler in (A3.5), second and third—the Foreign country budget constraint (A3.6) at t and $t+1$ respectively, and the last equation is the Foreign goods market clearing in (A3.7). R_t is exogenous to Foreign and given by (A3.16). Using that $P_t C_t = Y$, we first combine (A3.17)—(A3.19) to solve for $P_t^* C_t^*$

$$P_t^* C_t^* = Y(1 - \beta^s + R_t^{-1}), \tag{A3.21}$$

and then substitute this expression into (A3.20) to yield

$$Q_t = P_t^*/P_t = P_{F,t}^{1-\omega} = \left[(1-\omega)(1-\beta^s + R_t^{-1}) + \omega\right]^{1-\omega}, \tag{A3.22}$$

which, combined with the interest rate expression in (A3.16), gives us the real exchange rate formula in Proposition 3(a):

$$Q_t \ (:= Q_t^{\text{NoZLB}}) = \left[(1-\omega)\left(1 - \beta^s\left(1 - \frac{(1-\chi)Y + \chi \bar{B}^H}{(1-\chi)Y + \chi \bar{B}^L}\right)\right) + \omega\right]^{1-\omega}. \tag{A3.23}$$

Because $\bar{B}^H > \bar{B}^L$, $Q_t^{\text{NoZLB}} > 1$, hence the real exchange rate depreciates.

(b) At the zero lower bound

Suppose that stabilising union-wide inflation would require $R_t < 1$. From equation (A3.16), this requires a shock large enough, such that

$$\beta^s \bar{B}^H - \bar{B}^L > \frac{(1-\chi)}{\chi}(1-\beta)\gamma Y = \bar{\zeta}.$$

With the union central bank unable to stabilise the Home economy, Home enters a recession and $Y_t < Y$.[23] This, in turn, puts downward pressure on prices in Home, and from (A3.10), $P_t = \gamma P_{t-1} = \gamma$. We further note that in period $t+1$, the economy is in steady state, hence $R_{t+1} = (\beta^s)^{-1}$, $P_{t+1} = P_t = \gamma$, and $Y_{t+1} = Y$.

[23] To see this, note that $Y_t = Y$ would require the interest rate to equal the value in (A3.16). Since this violates the zero lower bound constraint and $Y_t \leq Y$, it must be the case that $Y_t < Y$. To see precisely by how much output falls, see equation (4.4) in the main text, and the proof of Proposition 4.

We now turn to developments in Foreign. Suppose first that Foreign wages are fully flexible. This means Foreign remains at full employment from (A3.11), and $Y_t^* = Y$. Furthermore, we have $R_t = 1$ from (A3.9), $P_t C_t = \gamma Y_t$ (plugging $P_t = 1$ into A3.3), and $P_{t+1} C_{t+1} = \gamma Y$. Plugging these values into the 4x4 system of equations in (A3.17)—(A3.20) yields an expression for Foreign nominal consumption

$$P_t^* C_t^* = \gamma Y (1 + (1 - \beta^s) \frac{Y_t}{Y}).$$

Combining this with the market clearing condition in (A3.7) gives us an expression for the real exchange rate under flexible Foreign prices:

$$Q_t = \left[\frac{P_{F,t}}{P_t} \right]^{1-\omega} = \left[1 - \omega + (1 - (1-\omega)\beta^s) \frac{Y_t}{Y} \right]^{1-\omega} \tag{A3.24}$$

We refer the reader to equation (4.4) and the proof of Propostion 4 for the precise expression for Y_t.

Suppose now that Foreign wages are downwardly rigid, and furthermore, the rigidity is sufficiently high such that (A3.11) binds and hence $P_{F,t} = \gamma^*$. Then, trivially,

$$Q_t = \left[\frac{\gamma^*}{\gamma} \right]^{1-\omega}.$$

We now combine the cases of rigid and flexible wages in Foreign. From (A3.13), we know that Foreign price level falls to either the flex-price level, or the maximum amount permitted by downward wage rigidity:

$$P_{F,t} = \max \left(\gamma^*, 1 - \omega + (1 - (1-\omega)\beta^s) \frac{Y_t}{Y} \right),$$

Putting this expression into the real exchange rate formula in (3.10) yields

$$Q_t \ (:= Q_t^{\text{ZLB}}) = \max \left(\left[\frac{\gamma^*}{\gamma} \right]^{1-\omega}, \left[1 - \omega + (1 - (1-\omega)\beta^s) \frac{Y_t}{Y} \right]^{1-\omega} \right), \tag{A3.25}$$

which is the formula in Proposition 3(b).

To establish the inequality $Q_t^{\text{ZLB}} < Q_t^{\text{NoZLB}}$, we require that both parts of the max() operator above are smaller in magnitude than Q_t^{NoZLB}. This first of all requires that Foreign wages are not excessively rigid relative to Home:

$$\left[\frac{\gamma^*}{\gamma} \right]^{1-\omega} < Q_t^{\text{NoZLB}},$$

which from (A3.23) requires

$$\gamma^*/\gamma < 1 + \kappa, \text{ where}$$
$$\kappa = (1-\omega)\beta^s \left(\frac{(1-\chi)Y + \chi \bar{B}^H}{(1-\chi)Y + \chi \bar{B}^L} - 1 \right) > 0$$

Turning to the second part of the max() operator, since $Y_t < Y$ and $1-(1-\omega)\beta^s > 0$, this is bounded above by

$$\left[1-\omega+(1-(1-\omega)\beta^s)\,\frac{Y_t}{Y}\right]^{1-\omega} < [2-\omega-(1-\omega)\beta^s]^{1-\omega}\,.$$

Because $\beta^s\bar{B}^H - \bar{B}^L > \underline{\zeta}$, we know that the counterfactual $R_t < 1$ in (A3.16), and hence $R_t^{-1} > 1$. Applying this inequality to equation (A3.22), we get

$$Q_t^{\text{NoZLB}} > [2-\omega-(1-\omega)\beta^s]^{1-\omega}\,,$$

Therefore, the inequality is satisfied for both parts of the max() operator, and $Q_t^{\text{ZLB}} < Q_t^{\text{NoZLB}}$.

To get a real appreciation, we need both parts of the max() operator to be less than 1. For the first part, we simply require $\gamma^*/\gamma < 1$. For the second part, we require

$$\beta^s\bar{B}^H - \bar{B}^L > \frac{(1-\chi)}{\chi}\left(\frac{1-\beta}{1-(1-\omega)\beta}\right)\gamma Y = \bar{\zeta}$$

which is obtained by substituting in for Y_t in (A3.24) (using the formula in (4.4), derived in the Proposition 4 proof), and setting the resulting expression to be less than 1.

A4 Proposition 4 proof

We start by summarising some useful results shown in the main text, and in the proof of Proposition 3. At the zero lower bound, Home real incomes and output fall below potential, and Home prices fall by γ at t, and remain at that level at $t+1$.

$$\frac{W_t L_t}{P_t} = Y_t < Y$$
$$P_t = P_{t+1} = \gamma$$

Substituting the above into the borrowers' budget constraint in (A3.2), and the savers' Euler equation in (A3.1) gives

$$C_t^b = -\frac{\bar{B}^H - \bar{B}^L}{\gamma} + Y_t \tag{A4.1}$$

$$C_t^s = (\beta^s)^{-1} C_{t+1}^s \tag{A4.2}$$

Turning to savers' budget constraint (equation A3.15) at $t+1$, and knowing that $\bar{B}_t = \bar{B}^L$ in the new steady state, we get

$$C_{t+1}^s = Y + \frac{\chi}{1-\chi}\left(\frac{(1-\beta^s)\bar{B}^L}{\gamma}\right) \tag{A4.3}$$

Combining (A3.3) in (A3.4) yields

$$Y_t = \chi C_t^s + (1-\chi) C_t^b,$$

and substituting for saver and borrower consumption using equations (A4.1)—(A4.3) gives us the expression for output in (4.4):

$$Y_t = (\beta^s)^{-1} \left[Y - \frac{\chi}{1-\chi} \left(\frac{\beta \bar{B}^H - \bar{B}^L}{\gamma} \right) \right]$$

We also note that if $\beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$, $Y_t < Y$ above.

From the above equation, we can see that output Y_t falls with lower γ. This also means that real incomes fall, and from (A4.1), that borrower consumption falls—since real incomes Y_t are lower, and real debt repayments $(\bar{B}^H - \bar{B}^L)/\gamma$, which enter negatively, are higher. This completes the proof of Proposition 4(a). As a sidenote, from (A4.3) and (A4.2) we can see that saver consumption increases slightly with more flexibility and lower prices, because saver consumption becomes higher in the new steady state. This is because the long-run real debt levels are higher. However, this increase in saver consumption comes at the expense of the borrowers (both during the deleveraging period and in the new steady state), and is not enough to offset the fall in borrower consumption during the deleveraging period (because output falls, from 4.4).

We now turn to part (b) of Proposition 4. If the Foreign wage rigidity constraint is not binding, the real exchange rate is given by the expression in (A3.24). Then γ only enters this expression via Y_t. We can see that higher wage flexibility lowers output Y_t, and, since $1-(1-\omega)\beta^s > 0$, lowers the real exchange rate Q_t. If $\bar{\zeta} > \beta^s \bar{B}^H - \bar{B}^L > \underline{\zeta}$, $Q_t > 1$ and the real exchange rate depreciates by less. If $\beta^s \bar{B}^H - \bar{B}^L > \bar{\zeta}$, $Q_t < 1$, and the real exchange rate appreciates by more. This completes the proof.