EUROPEAN COMMISSION
EUROSTAT

Directorate C: Macro-economic statistics
Unit C-4: Price statistics. Purchasing Power Parities. Housing statistics

HICP recommendation on bridged overlap ([1])

I. The recommendations

Bridged overlap is an implicit quality adjustment method that can be applied in the HICP when a disappearing product-offer is replaced with a new product-offer. Bridged overlap implicitly imputes a price for the new product-offer in the previous period and thereby values the quality difference as the price difference between the old and new product-offers in that period. The main principle of the recommendation is to monitor and identify replacement situations in which the assumptions underlying bridged overlap are either significantly or systematically not satisfied and modify the replacement and quality adjustment procedures as appropriate.

Recommendation 1: Cases in which bridged overlap should not be used

It is recommended to avoid applying bridged overlap in the following situations, unless duly justified:

1. The last price of the replaced (old) product-offer is a reduced price.
2. The first price of the new product-offer is a reduced price.
3. The first price of the new product-offer is unusually high.
4. The matched sample of product-offers includes reduced or atypical prices, or shows a downward price trend during the product life cycle.

Recommendation 2: Impact assessment of bridged overlap

It is recommended to regularly analyse the impact of bridged overlap on indices for relevant product groups over longer periods. Such an analysis can be conducted, for example and where possible, by comparing bridged overlap with other quality adjustment methods such as direct comparison, link-to-show-no-price change, or hedonics.

Recommendation 3: Alternatives to bridged overlap

When bridged overlap is considered inappropriate, another quality adjustment method should be used instead. There is no single best alternative to bridged overlap and the choice depends on circumstances. In practice, the following treatments could be considered: apply direct comparison, implement explicit quality adjustment procedures, adjust the products that enter the bridge, impute a normal price following a reduced price, optimise the timing of replacements, or apply bridged overlap with respect to a previous period.

([1]) This recommendation was endorsed by the Directors of Macro-Economic Statistics, in June 2021.

II. Explanatory text

1. Description of bridged overlap

When a product-offer becomes permanently missing, a replacement product-offer must be selected. If there is a quality difference between the replaced (old) and replacement (new) product-offer, a quality adjustment method must be applied ([2]). Bridged overlap ([3]) is an implicit quality adjustment method that estimates the pure price change component of the price difference between the old and new product-offers based on the price changes observed for similar product-offers. The difference between the pure price change and the observed price change is considered as change due to quality difference. As with any overlap method, the quality difference is implicitly measured by the ratio of prices of the old and new products in the common, overlapping time period.

Table 1 illustrates the application of bridged overlap. The product-offer A is available until t3 and is replaced by product-offer B in t4. The other product-offers (C-E) are available in all periods.

Table 1: Illustration of the bridged overlap method

	t0	t1	t2	t3	t4	t4/t3
Product-offer A	60.00	54.00	53.00	50.00		
Product-offer B				52.76	58.00	9.92%
Product-offer C	50.00	45.00	40.00	40.00	42.00	5.00%
Product-offer D	40.00	32.00	30.00	30.00	33.00	10.00%
Product-offer E	35.00	30.00	25.00	20.00	23.00	15.00%
Avg. price C-E (Bridge)				**28.84**	**31.71**	**9.92%**
Avg. price A, C-E	45.27	39.08	35.51	33.10		
Avg. price B-E				33.55	36.87	9.92%
Index t/t-1		86.33	90.86	93.21	109.92	
Index 0/t	100	86.33	78.44	73.11	80.37	

The aggregate price change up to period t3 is based on product-offers A, and C-E. Between t3 and t4, the prices of the matched product-offers (models C-E) are used to estimate a pure price change between product-offers A and B. The (geometric) average price of product-offers C-E stands at 28.84 in t3 and at 31.71 in t4. This corresponds to a price change of 9.92%. Therefore, the pure price change between product-offers A and B is also supposed to be 9.92%.

The bridged overlap method implicitly assumes that product-offer B would have had a price of 58/(1+9.92%)=52.76 in period t3 (backward imputation). The price difference in the overlap period between the old and new model measures the quality difference. For example, the estimated price for product-offer B in period t3 is 52.76 whereas the observed price for product-offer A in the same period is 50. This means that product-offer B is of 52.76/50-1 = 5.5% 'better quality' than product-offer A.

([2]) See Article 11(1) of Regulation (EU) 2020/1148. *If there is no quality difference between a replaced product and its replacement, Member States shall compare the observed prices directly. Otherwise, Member States shall make a quality adjustment.*

([3]) Bridged overlap is presented in section 6.8.1 in the HICP Methodological Manual.

The non-adjusted price change between product-offers A and B corresponds to 58/50-1=16%. This non-adjusted price change can thus be decomposed into a pure price change and a quality change:

$$\underbrace{\frac{58.00}{50.00}}_{\substack{non-adjusted\\ price\ change\\ =\mathbf{1.160}}} = \underbrace{\frac{58.00}{52.76}}_{\substack{pure\\ price\ change\\ =\mathbf{1.099}}} \cdot \underbrace{\frac{52.76}{50.00}}_{\substack{quality\\ change\\ =\mathbf{1.055}}}$$

For convenience, the bridged overlap method is presented here in terms of short term (month-on-month) price developments. This is because the bridged overlap method usually relies on price changes between two consecutive months. In practice, the prices may be compared to the price reference period (December of the previous year). The cumulative price change with respect to the price reference period can be obtained by multiplying the short-term price indices. The presentation is based on geometric average prices. One could adopt either an arithmetic average of prices (Dutot) or a geometric average of prices (Jevons). Depending on compilation systems in place at the national level, there may be different ways to implement bridged overlap ([4]). Bridged overlap as described in this document refers to prices of product-offers ([5]). Bridged overlap may also be relevant in the context of scanner data if a fixed basket methodology is implemented and new products are incorporated into the index compilation as replacements of disappearing products.

In contrast to bridged overlap, there are the following two alternatives of handling replacements. On the one hand, the link-to-show-no-price-change (LNP) method assumes that the pure price change between product-offer A available in t3 and product-offer B available in t4 is zero (see table 2). In other words, the price of the new product-offer is carried backwards, and the price difference between the new and old models in t3 is all due to quality difference. This method should only be used if duly justified, because it artificially biases the index towards no price change. On the other hand, the direct comparison (DC) method simply compares the price of product-offer A in t3 with the price of product-offer B in t4. This approach assumes that there is no quality difference between the two product-offers (see table 3).

Table 2: Illustration of the link-to-show-no-price-change method

	t0	t1	t2	t3	t4	t4/t3
Product-offer A	60.00	54.00	53.00	50.00		
Product-offer B				58.00	58.00	0.00%
Product-offer C	50.00	45.00	40.00	40.00	42.00	5.00%
Product-offer D	40.00	32.00	30.00	30.00	33.00	10.00%
Product-offer E	35.00	30.00	25.00	20.00	23.00	15.00%
Avg. price A, C - E	45.27	39.08	35.51	33.10		
Avg. price B - E				34.35	36.87	7.35%
Index t/t-1		86.33	90.86	93.21	107.35	
Index 0/t	100	86.33	78.44	73.11	78.49	

([4]) See paragraphs 8.66-8.76 in the CPI Manual Concepts and Methods (2020) for different implementations of Bridged Overlap depending on the index formula (Jevons or Dutot) and type of index (chained or direct).

([5]) See Article 2 of Regulation (EU) 2020/1148: *'product-offer' means a product specified by its characteristics, the timing and place of purchase and the terms of supply, and for which a price is observed.* With scanner data, products are typically defined as a set of product-offers amongst which there are no significant quality differences and for which an average price is calculated.

In the LNP method, the non-adjusted price change can be decomposed as follows. As there is no price change, everything is considered to be quality change ([6]).

$$\underbrace{\frac{58.00}{50.00}}_{\substack{non-adjusted\\ price\ change\\ =\mathbf{1.160}}} = \underbrace{\frac{58.00}{58.00}}_{\substack{pure\\ price\ change\\ =\mathbf{1.000}}} \cdot \underbrace{\frac{58.00}{50.00}}_{\substack{quality\\ change\\ =\mathbf{1.160}}}$$

Table 3: Illustration of the direct comparison method

	t0	t1	t2	t3	t4	t4/t3
Product-offer A	60.00	54.00	53.00	50.00		
Product-offer B				50.00	58.00	16.00%
Product-offer C	50.00	45.00	40.00	40.00	42.00	5.00%
Product-offer D	40.00	32.00	30.00	30.00	33.00	10.00%
Product-offer E	35.00	30.00	25.00	20.00	23.00	15.00%
Avg. price A, C-E	45.27	39.08	35.51	33.10		
Avg. price B-E				33.10	36.87	11.41%
Index t/t-1		86.33	90.86	93.21	111.41	
Index 0/t	100	86.33	78.44	73.11	81.45	

In the DC method, the non-adjusted price change can be decomposed as follows. As there is no quality change, everything is considered to be pure price change.

$$\underbrace{\frac{58.00}{50.00}}_{\substack{non-adjusted\\ price\ change\\ =\mathbf{1.160}}} = \underbrace{\frac{58.00}{50.00}}_{\substack{pure\\ price\ change\\ =\mathbf{1.160}}} \cdot \underbrace{\frac{50.00}{50.00}}_{\substack{quality\\ change\\ =\mathbf{1.000}}}$$

([6]) The LNP should in principle not be used in the HICP, unless duly justified. See Article 11(2) of Regulation (EU) 2020/1148: *Member States shall make a quality adjustment equal to the whole price difference between the replaced product in month m-1 and its replacement in month m only if this can be justified as an appropriate estimate of the quality difference.*

2. Assumptions underlying bridged overlap

The bridged overlap method ([7]) assumes that the quality-adjusted price change between the old and the new product-offer corresponds to the price change of the matched product-offers ('rate of change of the bridge'). We denote by p_n^{t-1} the price of the old product-offer n in period t-1 and by p_{n*}^t the price of the new product-offer $n*$ in period t. The price change obtained from the bridge is denoted by r_{Bridge} and the quality difference between product-offers n and $n*$ is $\widehat{\alpha_n}$. These variables are linked together as follows:

$$\frac{p_{n*}^t}{p_n^{t-1}\widehat{\alpha_n}} = r_{Bridge}$$

This means that the quality adjustment factor is implicitly defined as follows when bridged overlap is applied:

$$\widehat{\alpha_n} = \frac{p_{n*}^t}{p_n^{t-1} r_{Bridge}}$$

Bridged overlap relies on the idea that the prices of the product-offers that are sampled within an elementary aggregate react to each other instantaneously. When a price difference is observed it must arise from a difference in quality, such as product characteristics, timing, location, or conditions. In bridged overlap this condition is inherently used both for the quality difference between the replaced product-offer and the bridge in the preceding month, and for the quality difference between the bridge and the replacing product offer in the current month. Moreover, the prices of the product-offers included in the bridge should not be influenced by unusual price fluctuations ([8]).

Bridged overlap fails if the true (unobserved) quality adjustment factor α_n is either systematically or significantly different from $p_{n*}^t/(p_n^{t-1} r_{Bridge})$. The bias of the bridged overlap can be formalised as follows (see Annex 1):

$$Bias = \left(\frac{\alpha_n}{\widehat{\alpha_n}}\right)^{\frac{1}{n}}$$

([7]) Sections 2 and 3 rely on material from chapter 6 of the CPI Manual Concepts and Methods (2020). In that manual, bridged overlap is referred to as 'overall-mean imputation'.

([8]) In particular, items on sale and sales returns should be excluded. See example 4 in Annex 2.

3. Recommendations

It is recommended to constantly monitor and identify replacement situations in which the underlying assumptions of bridged overlap are either significantly or systematically not satisfied and modify the replacement and quality adjustment procedures where feasible.

3.1. Recommendation 1: Cases in which bridged overlap should not be used

Bridged overlap should be avoided in those cases where the underlying assumptions are significantly violated. In particular, bridged overlap can be problematic in the following circumstances:

1. The last price of the replaced (old) product-offer is a reduced price. This situation can be encountered at the end of a sales period and is especially common in clothing and footwear. Reduced prices can also be observed in situations of inventory clearing or closure of an outlet.

2. The first price of the new product-offer is a reduced price. This may happen at the beginning of the life cycle of a product under a market penetration strategy (setting a low initial price for a new model in order to attract a large number of buyers quickly).

3. The first price of the new product-offer is unusually high. This may happen at the beginning of the life cycle of a product under a price skimming strategy (setting a high initial price that a subset of customers is willing to pay in order to maximise profit).

4. The bridge is impacted by reduced or atypical prices or shows a downward price trend during the product life cycle, and these fluctuations do not apply to the replacement situation.

The following table includes some examples and possible biases that appear as a result of applying bridged overlap in such circumstances. Numerical illustrations of these examples can be found in Annex 2.

Table 4: Examples and bias of bridged overlap

	Example	Parameters (Stylised values)	Direction of bias
1	The last price of the replaced (old) product-offer is a reduced price	$p_{n*}^{t} \gg p_{n}^{t-1}$ $r_{Bridge}\approx 1$	Downward
2	The first price of the replacement (new) product-offer is a reduced price	$p_{n*}^{t} \ll p_{n}^{t-1}$ $r_{Bridge}\approx 1$	Upward
3	The first price of the replacement (new) product-offer is an unusually high price	$p_{n*}^{t} \gg p_{n}^{t-1}$ $r_{Bridge}\approx 1$	Downward
4	Some of the matched product-offers included in the bridge are reduced or atypical prices	$p_{n*}^{t} \geq p_{n}^{t-1}$ $r_{Bridge}<<1$	Downward

The application of bridged overlap is always an interplay between the price of the replaced and replacement product-offer, and of the bridge. Depending on the market circumstances and the reaction of the prices in the sample to new and disappearing products, the underlying assumptions of bridged overlap may or may not be satisfied. Bridged overlap can still be acceptable even when the replaced product-offer exits the sample with a reduced price (see Example 1). For example, in clothing, sales may occur in a synchronised manner across all stores. In such a case, the bridge is capturing a price increase as most prices were on sales in t-1 but not in t ($r_{Bridge} > 1$). This may still yield an implicit quality adjustment factor that remains reasonable. Bridged overlap may still work even if the new product enters at a low price (see Example 2). If the matched products react to the new entrant, their prices may go down as well ($r_{Bridge} < 1$), which mitigates possible biases.

In order to identify problematic replacements situations, the implicit quality adjustment factors of bridged overlap can be compared to some extreme threshold values, or to the quality adjustment factors implied by the direct comparison or link-to-show-no-price change methods (see Annex 4).

In markets with high product turnover, a method called monthly chaining and replenishment ([9]) is sometimes applied. This method is similar to bridged overlap as it relies on the price change of the matched product-offers to impute a price for the new product-offer in the previous month, or a price for the disappearing product-offer in the current month. Although it depends on the pricing strategies adopted by the sellers of these products, there may be similar downward biases as those outlined in example 1. For example, a downward bias may occur if the product-offers systematically exit the market at a discounted price.

3.2. Recommendation 2: Impact assessment of bridged overlap

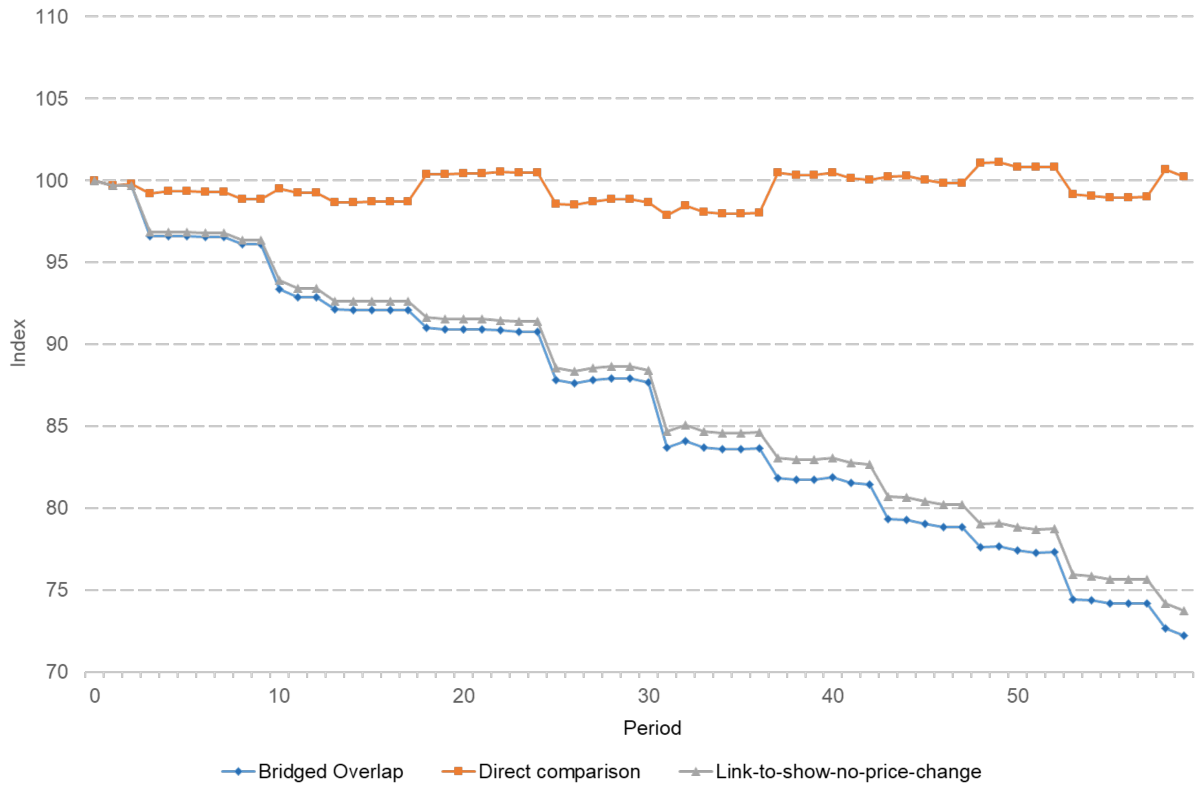
For most products, the bias tends to be in the same direction (most commonly downward) month after month. In any one month it does not make much difference, but when accumulated over a long period it can cause extreme results. In order to identify such situations and improve the replacement and quality adjustment procedures, a more structural analysis over a longer period is needed. This is especially relevant in product categories with a high number of bridged overlap replacements compared to sample size. It may also apply to product categories with price trends that appear to be less consistent with other indicators.

Such an analysis can be conducted by comparing the use of bridged overlap with other quality adjustment methods. For example, instead of applying bridged overlap, replacements could be handled either through direct comparison or link-to-show-no-price-change. In principle, it should be possible to conduct such an analysis without further data for example on product characteristics. If product characteristics are available, it can be possible to compare bridged overlap methods with hedonic index. Although hedonic methods have their own challenges, such an analysis could give some further insights on possible biases that result from the bridged overlap.

([9]) Monthly Chaining and Replenishment is presented in section 6.8.2 in the HICP Methodological Manual.

Figure 1 shows an example of comparing bridged overlap with direct comparison and link-to-show-no-price change. This study ([10]) is based on web scraped data from two German online supermarkets for beer. It shows that bridged overlap lies below link-to-show-no-price change, which both have significant downward biases compared to direct comparison. Figure 2 compares a bridged overlap price index with a hedonic price index for laptops/desktops. This study has been conducted by the Irish Central Statistics Office ([11]). Figure 3 analyses the impact of bridged overlap for men's garments. An alternative index is simulated where bridged overlap is applied on the pre-sales price and where direct comparison is more often used.

Figure 1: Impact of bridged overlap (ECB)



([10]) Bernhard Goldhammer, Raffaella Traverso, Lukas Henkel, 2019. Bias related to the bridged-overlap method. Poster presented at the 16th meeting of the Ottawa group, Rio de Janeiro, Brazil.

([11]) Joseph Keating, Matt Murtagh (2018). Quality adjustment in the Irish CPI. Paper presented at the UNECE Group of Experts on Consumer Price Indices, Geneva.

Figure 2: Impact of bridged overlap for laptops/desktops (CSO Ireland)

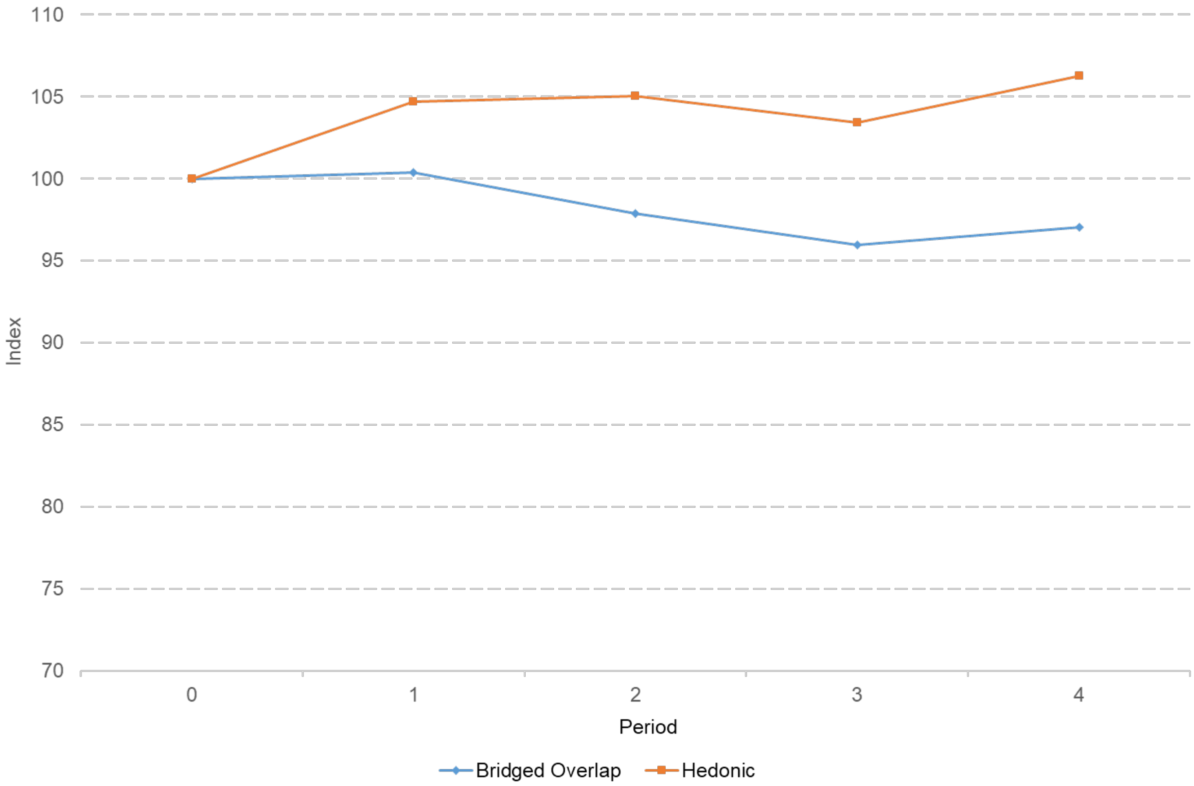
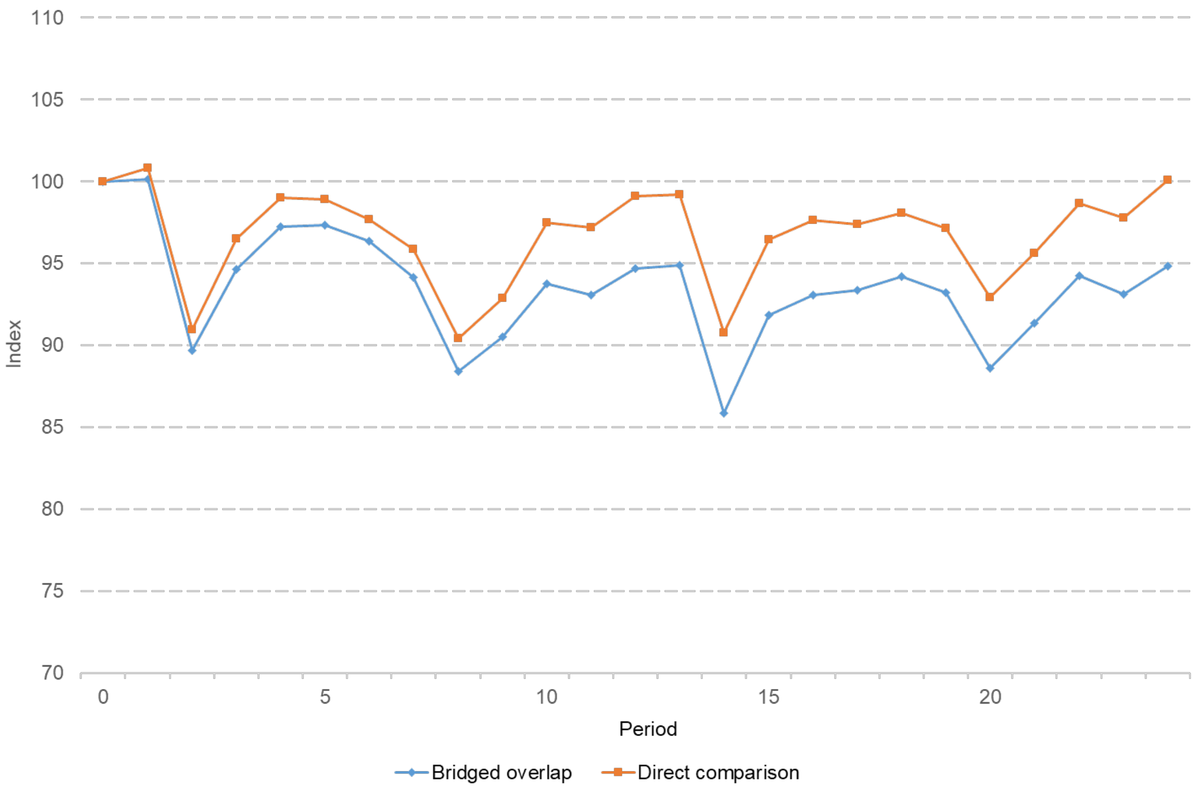


Figure 3: Impact of bridged overlap for men's garments (CSO Ireland)



3.3. Recommendation 3: Alternatives to bridged overlap

In order to avoid the biases caused by bridged overlap, the following alternative treatments can be considered instead:

- **Apply direct comparison**. The price of the new product-offer is directly compared to the price of the replaced product-offer. The application of DC assumes that price collection procedures are adapted to ensure that the replacement product is selected to be comparable to the old product (for instance the new product is selected within a tightly defined stratum). Moreover, a systematic and regular (e.g. annual) review of the sample must be conducted in order to ensure that the sample remains representative over time.

- **Apply explicit quality adjustment methods**. In these methods, a direct estimate of the value of the quality difference between the old and new product-offer is made. This estimation can for example be based on a hedonic function. Alternative methods can be option pricing or expert judgement.

- **Adjust the products that enter the bridge**. Not all matched product-offers within an elementary aggregate are used, but only a subset of them. In particular, product-offers with unusual price changes could be excluded. In practice, this approach can only be implemented if the number of product-offers is sufficiently large within an elementary aggregate so that a meaningful selection can be made that allows estimating a reliable average price change. Another variant of bridged overlap consists in estimating the price change between the old and new product based on the average price change derived from the price changes of comparable (direct price comparison) or explicitly adjusted replacements only. This means that only the prices of comparable and, where appropriate, explicitly quality-adjusted products are included in the bridge.

In addition, the following alternative treatments can be considered when the last observed price is a reduced price:

- **Go back to the last 'normal' price**. This means that the replacement is done one month later. In the first month following the disappearance, a normal price is used in the index compilation. In the second month following the disappearance, a replacement product is integrated using the standard techniques (for instance bridged overlap). It may not always be straightforward to identify the last normal price. One practical approach consists in using the price observed in the period prior to the period with a reduced price. Another approach consists in using an average (for example 3 or 6 months) of the prices observed prior to the period with a reduced price.

- **Optimise the timing of replacements**. The timing of the replacement is important as it can heavily influence the underlying implicit quality adjustment. This is especially important for products with short life-cycles. Where possible, a product can already be replaced prior to its last available period during which it may be sold at an unusually low price and with a low sales volume.

- **Apply bridged overlap between the current period and the period prior to the price reduction**. Bridged overlap is typically conducted with respect to the short-term monthly price change. A variant would be to apply the bridge between the current period and the period prior to the price reduction (for example between periods t and t-2). This variant of bridged overlap is illustrated in the Annex 3.

Annex 1: Formalisation of the bias

Suppose that we would know the 'true' quality adjusted price change between the old and the new product-offer. We could then use this true price change to compile an overall price change for the elementary aggregate. The difference between such a result and the price change obtained from a matched model approach corresponds to the bias of the bridged overlap method.

Let us suppose that the sample is composed of n product-offers ([12]). The products 1 to n-1 are available both in t-1 and in t. The individual product-offer n, for which a price is collected in t-1, is replaced by another product-offer n*, for which a price is collected in t. The pure price change for product-offer n* is obtained by comparing its observed price in period t to the quality adjusted price of product-offer n in t-1. This is the true (unobserved) pure price change:

$$r_{n*} = \frac{p_{n*}^{t}}{p_{n}^{t-1}\alpha_{n}}$$

In the bridged overlap method, we look at the price change obtained from the remaining n-1 matched product-offers:

$$r_{Bridge} = \prod_{i=1}^{n-1} \left(\frac{p_{i}^{t}}{p_{i}^{t-1}}\right)^{\frac{1}{n-1}} = \frac{p_{n*}^{t}}{p_{n}^{t-1}\widehat{\alpha_{n}}}$$

The difference of the bridged overlap index compared to the true index can be expressed as follows:

$$BIAS = \prod_{i=1}^{n-1} \left(\frac{p_{i}^{t}}{p_{i}^{t-1}} \frac{p_{n*}^{t}}{p_{n}^{t-1}\widehat{\alpha_{n}}}\right)^{\frac{1}{n}} \Bigg/ \prod_{i=1}^{n-1} \left(\frac{p_{i}^{t}}{p_{i}^{t-1}} \frac{p_{n*}^{t}}{p_{n}^{t-1}\alpha_{n}}\right)^{\frac{1}{n}}$$

This equation can be rewritten as follows:

$$BIAS = \left(\frac{p_{n*}^{t}/(p_{n}^{t-1}\widehat{\alpha_{n}})}{p_{n*}^{t}/(p_{n}^{t-1}\alpha_{n})}\right)^{\frac{1}{n}} = \left(\frac{\alpha_{n}}{\widehat{\alpha_{n}}}\right)^{\frac{1}{n}}$$

([12]) We suppose that only 1 out of the n available product-offers must be replaced, which means there are n-1 matched product-offers. The calculations can be further generalised for the case of m replacements and n-m matched product-offers.

Annex 2: Numerical illustrations

Example 1: The last price of the replaced (old) product-offer is a reduced price

	t0	t1	t2	t3	t4	t4/t3
Product-offer A	60.00	54.00	53.00	25.00		
Product-offer B				52.76	58.00	9.92%
Product-offer C	50.00	45.00	40.00	40.00	42.00	5.00%
Product-offer D	40.00	32.00	30.00	30.00	33.00	10.00%
Product-offer E	35.00	30.00	25.00	20.00	23.00	15.00%
Avg. price C-E (Bridge)				**28.84**	**31.71**	**9.92%**
Avg. price A, C-E	45.27	39.08	35.51	27.83		
Avg. price B-E				33.55	36.87	9.92%
Index t/t-1		86.33	90.86	78.38	109.92	
Index 0/t	100	86.33	78.44	61.48	67.58	

$$\underbrace{\frac{58.00}{25.00}}_{\substack{non-adjusted\\ price\ change\\ =\mathbf{2.320}}} = \underbrace{\frac{58.00}{52.76}}_{\substack{pure\\ price\ change\\ =\mathbf{1.099}}} \cdot \underbrace{\frac{52.76}{25.00}}_{\substack{quality\\ change\\ =\mathbf{2.111}}}$$

Example 2: The first price of the replacement (new) product-offer is a reduced price

	t0	t1	t2	t3	t4	t4/t3
Product-offer A	60.00	54.00	53.00	53.00		
Product-offer B				22.74	25.00	9.92%
Product-offer C	50.00	45.00	40.00	40.00	42.00	5.00%
Product-offer D	40.00	32.00	30.00	30.00	33.00	10.00%
Product-offer E	35.00	30.00	25.00	20.00	23.00	15.00%
Avg. price C-E (Bridge)				**28.84**	**31.71**	9.92%
Avg. price A, C-E	45.27	39.08	35.51	33.58		
Avg. price B-E				27.18	29.88	9.92%
Index t/t-1		86.33	90.86	94.57	109.92	
Index 0/t	100	86.33	78.44	74.18	81.55	

$$\underbrace{\frac{25.00}{53.00}}_{\substack{non-adjusted\\ price\ change\\ =\mathbf{0.472}}} = \underbrace{\frac{25.00}{22.74}}_{\substack{pure\\ price\ change\\ =\mathbf{1.099}}} \cdot \underbrace{\frac{22.74}{53.00}}_{\substack{quality\\ change\\ =\mathbf{0.429}}}$$

Example 3: The first price of the replacement (new) product-offer is an unusually high price

	t0	t1	t2	t3	t4	t4/t3
Product-offer A	60.00	60.00	60.00	60.00		
Product-offer B				120.00	120.00	0.00%
Product-offer C	50.00	45.00	40.00	40.00	40.00	0.00%
Product-offer D	40.00	32.00	30.00	30.00	30.00	0.00%
Product-offer E	35.00	30.00	25.00	20.00	20.00	0.00%
Avg. price C-E (Bridge)				**28.84**	**28.84**	**0.00%**
Avg. price A, C-E	45.27	40.12	36.63	34.64		
Avg. price B-E				41.20	41.20	0.00%
Index t/t-1		88.63	91.29	94.57	100.00	
Index 0/t	100	88.63	80.91	76.52	76.52	0.00%

$$\underbrace{\frac{120}{60}}_{\substack{non-adjusted\\ price\ change\\ =\mathbf{2.000}}} = \underbrace{\frac{120}{120}}_{\substack{pure\\ price\ change\\ =\mathbf{1.000}}} \cdot \underbrace{\frac{120}{60}}_{\substack{quality\\ change\\ =\mathbf{2.000}}}$$

Example 4: Some of the matched product-offers included in the bridge are reduced or atypical prices

	t0	t1	t2	t3	t4	t4/t3
Product-offer A	60.00	54.00	53.00	53.00		
Product-offer B				67.57	58.00	-14.16%
Product-offer C	50.00	45.00	40.00	40.00	20.00	-50.00%
Product-offer D	40.00	32.00	30.00	30.00	33.00	10.00%
Product-offer E	35.00	30.00	25.00	20.00	23.00	15.00%
Avg. price C-E (Bridge)				**28.84**	**24.76**	**-14.16%**
Avg. price A. C-E	45.27	39.08	35.51	33.58		
Avg. price B-E				35.69	30.63	-14.16%
Index t/t-1		86.33	90.86	94.57	85.84	
Index 0/t	100	86.33	78.44	74.18	63.68	

$$\underbrace{\frac{58.00}{53.00}}_{\substack{non-adjusted\\ price\ change\\ =\mathbf{1.094}}} = \underbrace{\frac{58.00}{67.57}}_{\substack{pure\\ price\ change\\ =\mathbf{0.858}}} \cdot \underbrace{\frac{67.57}{53.00}}_{\substack{quality\\ change\\ =\mathbf{1.275}}}$$

Annex 3: Variants of bridged overlap

In the following example, the bridge was applied between period t4 (current period) and period t2 (the period prior to the price reduction). Any other, earlier, period, in particular the first period t0 could also be used as a starting point for the bridge.

Bridged overlap is applied between the current period and the period prior to the price reduction

	t0	t1	t2	t3	t4	t4/t3
Product-offer A	60.00	54.00	53.00	25.00 (price reduction)		
Product-offer B			58/(1+3.87%) = 55.84		58.00	
Product-offer C	50.00	55.00	55.00	56.00	57.00	
Product-offer D	40.00	40.00	41.00	42.00	42.00	
Product-offer E	35.00	36.00	36.00	37.00	38.00	
Avg. price C-E (Bridge)			**43.30**		**44.97**	3.87%
Avg. price A, C-E	45.27	45.48	45.54	38.41		
Avg. Price B-E			46.14		47.93	
Index t/t-1		100.45	100.15	84.33		
Index t/t-2					103.87	
Index 0/t	100	100.45	100.60	84.84	104.50 =100.6* 103.87/100	

Annex 4: Critical values for a plausibility check for bridged overlap results

The formalisation of bridged overlap, presented in this document, together with some considerations about the relationship between link-to-show-no price change and direct price comparison to bridged overlap can yield some critical values of bridged overlap results. These critical values can be used for plausibility checks in HICP production. Values outside these boundaries do not necessarily imply that the bridged overlap method is incorrect.

We start with the formula that creates a relationship between the price of the old product in the previous month, the quality adjustment factor, the bridge and the collected price of the new product in the current month:

$$\frac{p_{n*}^{t}}{p_n^{t-1}\widehat{\alpha_n}} = r_{Bridge} \quad (1)$$

This can be re-written as:

$$p_{n*}^{t} = p_n^{t-1}\widehat{\alpha_n}r_{Bridge} \quad (2)$$

So, the price of the new product is explained by the price of the old product in period t-1, the quality adjustment factor, and the bridge which represents inflation. As we adjust for quality, an equal estimate for the price of the old product in period t is given as ([13]):

$$\widehat{p_n^t} \equiv \frac{p_{n*}^{t}}{\widehat{\alpha_n}} = p_n^{t-1}r_{Bridge} \quad (3)$$

Let us think about implicit quality adjustment methods and assume that $p_{n*}^{t} > p_n^{t-1}$. Under normal circumstances, we would expect the price change of bridged overlap being somewhere in between DC and LNP. We get a continuum of methods:

- **Direct price comparison** is the one possible case that can be easily calculated: the quality difference Δq is assumed to be 0, the price difference is totally seen as price change Δp. From formula (2) follows $\hat{\alpha}_{n,DC}$ =1 and therefore:

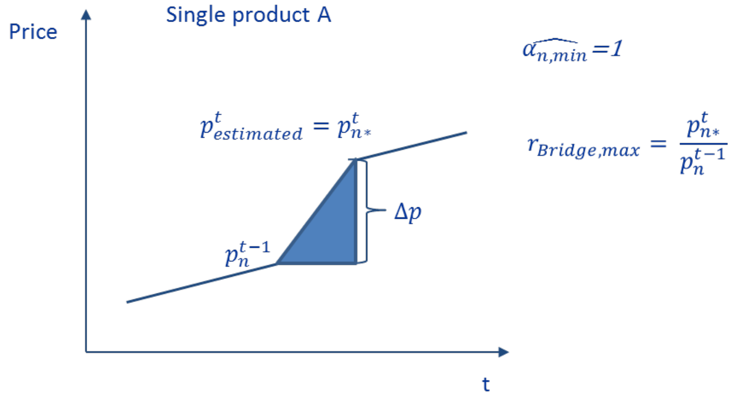
$$p_{n*}^{t} = p_n^{t-1}r_{DC} \quad (4)$$

The 'bridge factor' is just the ratio of the prices. It can be seen as an upper bound for the bridge factor of bridged overlap:

$$r_{DC} = \frac{p_{n*}^{t}}{p_n^{t-1}} \quad (5)$$

([13]) Please note that, for illustrative purposes (especially the graphical representations), the case that the price in the replacement period is adjusted is shown. The considerations are equally valid for the case where the price in the previous period is adjusted.

The following figure shows this extreme case which assumes no quality difference.



- **Link-to-show-no price change** is another possibility that can be easily calculated. All difference is explained with quality difference Δq, so, r_{LNP}=1. And the quality adjustment factor reaches its maximum:

$$p^t_{n*} = p^{t-1}_n \hat{\alpha}_{n,LNP} \tag{6}$$

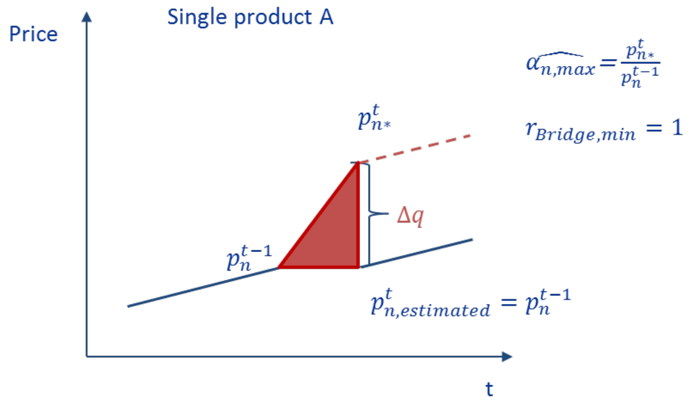
So, the upper bound for the quality adjustment factor is vice versa:

$$\hat{\alpha}_{n,LNP} = \frac{p^t_{n*}}{p^{t-1}_n} \tag{7}$$

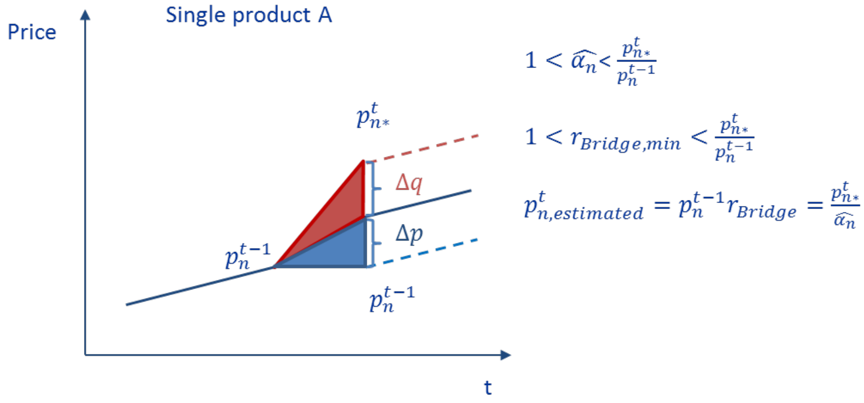
Using definition (3), the estimated price for the new product in t is given as

$$\widehat{p^t_n} = \frac{p^t_{n*}}{\hat{\alpha}_n} = p^t_{n*}\frac{p^{t-1}_n}{p^t_{n*}} = p^{t-1}_n \tag{8}$$

as assumed. A graphical representation of this case is given in the next figure:



- For **bridged overlap**, we would assume a situation like the following:



Both bridge factor and quality adjustment factor are between their minimum and maximum values derived from the extreme cases of DC and LNP, a decomposition of the nominal price change into quality-related price difference and real price difference takes place. Note that both Δq and Δp in the figure above have positive values.

From the above, it becomes clear that the critical value of both the quality adjustment factor and the bridge factor are 1 and $\frac{p_{n*}^t}{p_n^{t-1}}$.

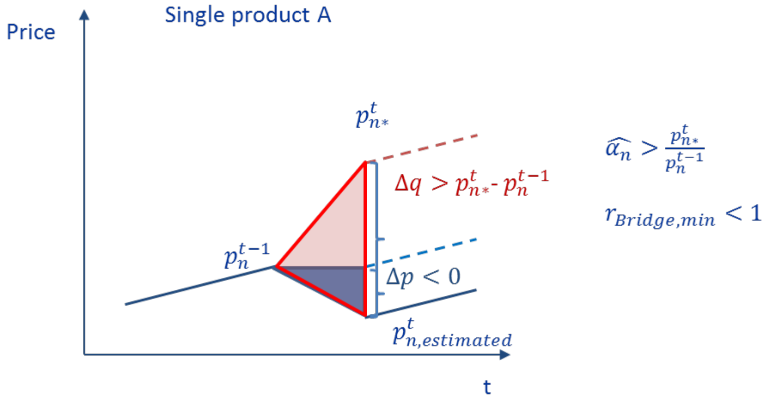
What happens if these values are exceeded? Let us consider one example. Let us assume that we have a bridge factor of 0.8, which corresponds to a price decrease in all available products in 20% (can happen for consumer electronics). From equation (3), we get for the quality adjustment factor:

$$\widehat{\alpha_n} = \frac{p_{n*}^t}{0.8 p_n^{t-1}} = 1.25 \frac{p_{n*}^t}{p_n^{t-1}} > \frac{p_{n*}^t}{p_n^{t-1}}$$

which is also outside the interval $\left[1; \frac{p_{n*}^t}{p_n^{t-1}}\right]$ From equation (3), we can also see that:

$$\widehat{p_n^t} = 0.8 p_n^{t-1}$$

Which means that Δp is negative and Δq is larger than the difference between p_{n*}^t and p_n^{t-1}. In a graphical illustration, this looks as follows:



If we assume that DC and LNP are two cases that can be easily understood and calculated, then such a result would clearly need an investigation whether such a quality adjustment that turns a nominal increase in a quality-adjusted decrease is justified. Therefore, plausibility checks about the size of the bridge factor which should be between 1 and $\frac{p_{n*}^t}{p_n^{t-1}}$ could improve the performance of quality adjustment and, therefore, enhance the quality of the index as such. Even if the values are outside these anchors, the quality adjustment may still be appropriate given the characteristics of the replaced and replacement product.

NB: The example also works with $p_n^{t-1} > p_{n*}^t$: but in this case, the factor boundaries are the other way round: $1 > r_{Bridge} > \frac{p_{n*}^t}{p_n^{t-1}}$. So, for a plausibility check, these two cases need to be differentiated.